MATH 240 **Sample Questions** for Exam 2 on 10/26/2007

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1. Let A be the following matrix. $$A = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & -3 & 2 & 14 \\ 1 & -5 & -1 & 8 \\ 3 & 1 & -1 & -2 \end{bmatrix}$$

(a) Find the reduced row echelon form of A.

(b) Find the rank and nullity of A.

(c) Find a basis for the row space of A.

(d) Find a basis for the column space of A.

(e) Find a basis for the nullspace of A.

2. Let M_{22} be the vector space of all 2×2 matrices, and let $Q = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

(a) Let W be the set of all matrices A in M_{22} such that $AQ = 0$. Show that W is a subspace of M_{22}.

(b) Find a basis for W, and find the dimension of W.

3. Let $S = \{(1,0,0), (1,1,0), (1,1,1)\}$ and $T = \{(1,-1,0), (0,1,-1), (0,0,1)\}$ be ordered bases for $\mathbf{R}_3$. Let $\mathbf{v} = (3,2,1)$.

(a) Find the coordinate vector $[\mathbf{v}]_T$ of $\mathbf{v}$ with respect to the basis T.

(b) Find the transition matrix $P_{S \leftarrow T}$.

(c) Use $P_{S \leftarrow T}$ to find the coordinate vector $[\mathbf{v}]_S$ of $\mathbf{v}$ with respect to the basis S.

4. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be linearly independent vectors in $\mathbf{R}^n$. Prove that if A is a nonsingular $n \times n$ matrix, then the vectors $\{A\mathbf{v}_1, A\mathbf{v}_2, A\mathbf{v}_3\}$ are linearly independent in $\mathbf{R}^n$.

5. Find $\det(A)$ by row-reducing A to an upper triangular matrix, for the matrix $A = \begin{bmatrix} 2 & 0 & -1 & 7 \\ 6 & 1 & 0 & 4 \\ 8 & -2 & 1 & 0 \\ 4 & 1 & 0 & 2 \end{bmatrix}$.

6. Show that $\begin{vmatrix} x-4 & 0 & -1 \\ 2 & x-1 & 0 \\ 2 & 0 & x-1 \end{vmatrix} = (x-1)(x-2)(x-3)$.

7. Suppose that A and B are similar matrices. Explain why $\det(A) = \det(B)$.

8. Find the adjoint of $A = \begin{bmatrix} a & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Check that $A \cdot \operatorname{adj}(A) = \det(A) \cdot I_3$.