MATH 420 TEST II NAME ________________________________

Prof. John Beachy July 11, 2005

1. (40 points) Fill in the blank

   (a) Definition. Let $f : S \to T$ be a function. Then f is called a *one-to-one* function if

   (b) Definition. Let $f : S \to T$ be a function. Then f is called a *onto* function if

   (c) Proposition. Let $f : S \to T$ be a function. Then f is one-to-one if and only if f is onto, provided that

   (d) Definition. Let $f : S \to T$ be a function. Then f has an *inverse function* if

   (e) Definition. Let S be a set, and let R be a subset of $S \times S$. Then R is called an *equivalence relation on* S if

   (f) Definition Let S be a set. A *partition of* S is

   (g) Definition. Let S be a set. A *permutation* of S is

   (h) Definition. Let σ be a permutation in S_n. The *order* of σ is

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2. (10 pts) Define the formula $f : \mathbf{Z}_{12} \to \mathbf{Z}_{12}$ by $f([x]_{12}) = [x]_{12}^2$, for all $[x]_{12} \in \mathbf{Z}_{12}$. Show that the formula f defines a function. Find the image of f and the set $\mathbf{Z}_{12}/f$ of equivalence classes determined by f.

   (10 pts) Let $f : S \to T$ and $g : T \to U$ be functions, and assume that f is onto. Show that the composition $g \circ f$ is onto if and only if g is onto.

3. (10 pts) On the set S of all $n \times n$ matrices with entries in $\mathbf{R}$, define $A \sim B$ if there exists an invertible matrix P such that $B = PAP^{-1}$. Show that $\sim$ is an equivalence relation.

   (10 pts) On the set $\mathbf{Z}$ of integers, define $m \sim n$ if there exist positive integers k, j with $n | m^k$ and $m | n^j$. Show that $\sim$ is an equivalence relation on $\mathbf{Z}$.

4. (10 pts) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 2 & 7 & 3 & 4 & 6 & 8 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 3 & 5 & 7 & 6 & 1 & 2 \end{pmatrix}$.
   Write each of σ, τ, $\sigma\tau$, and $\tau\sigma$ as a product of disjoint cycles, and find the order of each of these permutations.

   (10 pts) Let σ and τ be permutations in S_n. Show that $\tau\sigma$ has the same order as $\sigma\tau$.