Homework 1

due Friday, September 1, 2006

7. Let $a, b, c$ be integers. Give a proof for these facts about divisors:
   (a) If $b | a$, then $b | ac$.
   (b) If $b | a$ and $c | b$, then $c | a$.
   (c) If $c | a$ and $c | b$, then $c | (ma + nb)$ for any integers $m, n$.

10. Let $a, b, c$ be integers, with $c \neq 0$. Show that $bc | ac$ if and only if $b | a$.

16. Let $a, b, c$ be integers, with $b > 0, c > 0$, and let $q$ be the quotient and $r$ the remainder when $a$ is divided by $b$.
   (a) Show that $q$ is the quotient and $rc$ is the remainder when $ac$ is divided by $bc$.
   (b) Show that if $q'$ is the quotient when $q$ is divided by $c$, then $q'$ is the quotient when $a$ is divided by $bc$. (Do not assume that the remainders are zero.)

21. Prove that the sum of the cubes of any three consecutive positive integers is divisible by 3.

Quiz 1

on Friday, September 1, 2006

You need to know the following definitions and statements of results: 1.1.1 through 1.1.5.

Questions will be taken from the following list of problems: pages 13–14, #2, 8, 9, 15, 17, 18

2. Find the quotient and remainder when $a$ is divided by $b$.
   (a) $a = 99$, $b = 17$
   (b) $a = -99$, $b = 17$
   (c) $a = 17$, $b = 99$
   (d) $a = -1017$, $b = 99$

8. Let $a, b, c$ be integers such that $a + b + c = 0$. Show that if $n$ is an integer which is a divisor of two of the three integers, then it is also a divisor of the third.

9. Let $a, b, c$ be integers.
   (a) Show that if $b | a$ and $b | (a + c)$, then $b | c$.
   (b) Show that if $b | a$ and $b \not| c$, then $b \not| (a + c)$.

15. Give a detailed proof of the statement in the text that if $a$ and $b$ are integers, then $b | a$ if and only if $a\mathbb{Z} \subseteq b\mathbb{Z}$.

17. Let $a, b, n$ be integers with $n > 1$. Suppose that $a = nq_1 + r_1$ with $0 \leq r_1 < n$ and $b = nq_2 + r_2$ with $0 \leq r_2 < n$. Prove that $n | (a - b)$ if and only if $r_1 = r_2$.

18. Show that any nonempty set of integers that is closed under subtraction must also be closed under addition.