Homework 1 due Friday, September 1, 2006

Hand in: pages 14–15, #7, 10, 16, 21

7. Let a, b, c be integers. Give a proof for these facts about divisors:
   (a) If $b \mid a$, then $b \mid ac$.
   (b) If $b \mid a$ and $c \mid b$, then $c \mid a$.
   (c) If $c \mid a$ and $c \mid b$, then $c \mid (ma + nb)$ for any integers m, n.

10. Let a, b, c be integers, with $c \neq 0$. Show that $bc \mid ac$ if and only if $b \mid a$.

16. Let a, b, c be integers, with $b > 0, c > 0$, and let q be the quotient and r the remainder when a is divided by b.
   (a) Show that q is the quotient and rc is the remainder when ac is divided by bc.
   (b) Show that if q' is the quotient when q is divided by c, then q' is the quotient when a is divided by bc. (Do not assume that the remainders are zero.)

21. Prove that the sum of the cubes of any three consecutive positive integers is divisible by 3.

Quiz 1 on Friday, September 1, 2006

You need to know the following definitions and statements of results: 1.1.1 through 1.1.5.

Questions will be taken from the following list of problems: pages 13–14, #2, 8, 9, 15, 17, 18

2. Find the quotient and remainder when a is divided by b.
   (a) $a = 99\,, \quad b = 17$
   (b) $a = -99\,, \quad b = 17$
   (c) $a = 17\,, \quad b = 99$
   (d) $a = -1017\,, \quad b = 99$

8. Let a, b, c be integers such that $a + b + c = 0$. Show that if n is an integer which is a divisor of two of the three integers, then it is also a divisor of the third.

9. Let a, b, c be integers.
   (a) Show that if $b \mid a$ and $b \mid (a + c)$, then $b \mid c$.
   (b) Show that if $b \mid a$ and $b \nmid c$, then $b \nmid (a + c)$.

15. Give a detailed proof of the statement in the text that if a and b are integers, then $b \mid a$ if and only if $a\mathbf{Z} \subseteq b\mathbf{Z}$.

17. Let a, b, n be integers with $n > 1$. Suppose that $a = nq_1 + r_1$ with $0 \leq r_1 < n$ and $b = nq_2 + r_2$ with $0 \leq r_2 < n$. Prove that $n \mid (a - b)$ if and only if $r_1 = r_2$.

18. Show that any nonempty set of integers that is closed under subtraction must also be closed under addition.