MATH 420, Section 1 **Homework 2** due Friday, September 8, 2006
Prof. Beachy

Hand in: 1.1, #13, 20 and 1.2, #12, 14

1.1 #13. Show that if n is any integer, then $(a + nb, b) = (a, b)$.

1.1 #20. Let a and b be integers, not both zero. An integer d is called the greatest common divisor of the nonzero integers a and b if (i) $d\,|\,a$ and $d\,|\,b$, and (ii) $c\,|\,a$ and $c\,|\,b$ implies $d \geq c$. Show that this definition is equivalent to Definition 1.1.5.

1.2 #12. Let a, b, c be nonzero integers. Show that $(a, b) = 1$ and $(a, c) = 1$ if and only if $(a, [b, c]) = 1$.

1.2 #14. Let a, b be nonzero integers with $(a, b) = 1$. Compute $(a + b, a - b)$.

Quiz 2 on Monday, September 11, 2006

You need to know the following definitions and statements of results: 1.1.1 through 1.2.4 and 1.2.9.

Questions will be taken from the following list of problems: 1.1 #4a,c,e, 5a,c,e, 12, 1.2 #3,4,5,13, and the following problems from the Study Guide.

1.1. Find $\gcd(435, 377)$, and express it as a linear combination of 435 and 377.

1.1 Find $\gcd(3553, 527)$, and express it as a linear combination of 3553 and 527.

1.2 (a) Use the Euclidean algorithm to find $\gcd(1776, 1492)$.
(b) Use the prime factorizations of 1492 and 1776 to find $\gcd(1776, 1492)$.

1.2 For positive integers a, b, prove that $\gcd(a, b) = 1$ if and only if $\gcd(a^2, b^2) = 1$.

1.2 Prove that $n - 1$ and $2n - 1$ are relatively prime, for all integers $n > 1$. Is the same true for $2n - 1$ and $3n - 1$?