Questions from Test II, 4/4/1997:

1. (20 pts) In S_6, let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 1 & 2 & 6 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 6 & 5 & 1 \end{pmatrix}$.

   (a) Write each of σ, τ, and $\sigma\tau$ as a product of disjoint cycles.

   (b) Find the order of each of σ, τ, and $\sigma\tau$.

   (c) Find σ^{-1}, τ^{-1}, and $(\sigma\tau)^{-1}$.

   (d) Is $(\sigma\tau)^{-1} = \sigma^{-1}\tau^{-1}$?

2. (20 pts) Let $f : S \to T$ and $g : T \to U$ be functions.

   (a) State these definitions: f is *one-to-one*; f is *onto*.

   (b) Prove Proposition 2.1.6 from the text, which states that (i) if f and g are one-to-one, then so is the composition $g \circ f$, and (ii) if f and g are onto, then so is $g \circ f$.

Questions from TEST II, 3/28/2003

1. (15 pts) Define $f : \mathbf{Z}_{12} \to \mathbf{Z}_{18}$ by $f([x]_{12}) = [6x]_{18}$, for all $[x]_{12} \in \mathbf{Z}_{12}$.

   (a) Show that f is a well-defined function.

   Recall: you must show that if $a \equiv b \ (mod\, 12)$, then $6a \equiv 6b \ (mod\, 18)$.

   (b) Find the image $f(\mathbf{Z}_{12})$ and the set of equivalence classes $\mathbf{Z}_{12}/f$ defined by f, and exhibit the one-to-one correspondence between these sets.

2. (20 pts) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 2 & 7 & 3 & 4 & 6 & 8 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 3 & 5 & 7 & 6 & 1 & 2 \end{pmatrix}$.

   (a) Write each of σ, τ, $\sigma\tau$, $\tau\sigma$, and $\sigma\tau\sigma^{-1}$ as a product of disjoint cycles.

   (b) Find the order of each of σ, τ, $\sigma\tau$, $\tau\sigma$, and $\sigma\tau\sigma^{-1}$.

   Recall: the order of a permutation σ is the smallest positive exponent m for which σ^m is equal to the identity.

3. (15 pts) Let σ and τ be permutations in S_n.

   (a) Show that $\sigma\tau\sigma^{-1}$ has the same order as τ.

   (b) Show that $\tau\sigma$ has the same order as $\sigma\tau$.

Questions from Test II, 7/11/2005

1. Definition. Let S be a set, and let R be a subset of $S \times S$. Then R is called an *equivalence relation on S* if

2. Definition Let S be a set. A *partition of S* is

3. (10 pts) On the set S of all $n \times n$ matrices with entries in $\mathbf{R}$, define $A \sim B$ if there exists an invertible matrix P such that $B = PAP^{-1}$. Show that $\sim$ is an equivalence relation.