

Homework 1

due Thursday, June 21, 2007

Hand in:

From the textbook, pages 14–15, #7, 14, 16, 21

7. Let a, b, c be integers. Give a proof for these facts about divisors:
- (a) If $b \mid a$, then $b \mid ac$.
 - (b) If $b \mid a$ and $c \mid b$, then $c \mid a$.
 - (c) If $c \mid a$ and $c \mid b$, then $c \mid (ma + nb)$ for any integers m, n .
14. For what positive integers n is it true that $\gcd(n, n + 2) = 2$? Prove your claim.
16. Let a, b, c be integers, with $b > 0, c > 0$, and let q be the quotient and r the remainder when a is divided by b .
- (a) Show that q is the quotient and rc is the remainder when ac is divided by bc .
 - (b) Show that if q' is the quotient when q is divided by c , then q' is the quotient when a is divided by bc . (Do not assume that the remainders are zero.)
21. Prove that the sum of the cubes of any three consecutive positive integers is divisible by 3.

From the Study Guide: pages 8–9, #39, 40, 44, 45

39. Find the quotient and remainder when a is divided by b .
- (a) $a = 12345$, $b = 100$
 - (b) $a = -12345$, $b = 100$
 - (c) $a = 123$, $b = 9$
 - (d) $a = 12345$, $b = 9$
 - (e) $a = 7862$, $b = 9$
 - (f) $a = 123$, $b = 11$
 - (g) $a = 12345$, $b = 11$
 - (h) $a = 7862$, $b = 11$
40. Find $\gcd(252, 180)$ and write it as a linear combination of 252 and 180.
44. Find $\gcd(13651, 3179)$ and express it as a linear combination of 13651 and 3179.
45. Let a, b be positive integers. Prove that $\gcd(a, b) = 1$ if and only if $\gcd(a + b, ab) = 1$