Homework 3  

Hand in:

From the Study Guide: page 13, §1.3 #48, 49, 50, 54, 55

48. Solve the following congruence.  
   \[ 25x \equiv 45 \pmod{60} \]

49. Find the additive orders of each of the following elements, by solving the appropriate congruences#.
   (a) 4, 5, 6 modulo 24
   (b) 4, 5, 6 modulo 25

50. Find the additive orders of each of the following elements, by solving the appropriate congruences.
   (a) 7, 8, 9 modulo 24
   (b) 7, 8, 9 modulo 25

54. Solve the following system of congruences:
   \[ x \equiv 13 \pmod{25} \quad x \equiv 9 \pmod{18} \]

55. Solve the following system of congruences:
   \[ x \equiv 9 \pmod{25} \quad x \equiv 13 \pmod{18} \]

From the Study Guide: pages 15-16, §1.4 #45, 46, 47, 49, 50

45. Find the multiplicative inverses of the given elements (if possible).
   (a) [12] in \( \mathbb{Z}_{15} \)
   (b) [14] in \( \mathbb{Z}_{15} \)
   (c) [7] in \( \mathbb{Z}_{15} \)
   (d) [12] in \( \mathbb{Z}_{23} \)
   (e) [14] in \( \mathbb{Z}_{32} \)

46. Find the multiplicative orders of the following elements.
   (a) [5] and [7] in \( \mathbb{Z}_{16}^\times \)
   (b) [5] and [7] in \( \mathbb{Z}_{17}^\times \)
   (c) [5] and [7] in \( \mathbb{Z}_{18}^\times \)

47. Find the multiplicative order of each element of \( \mathbb{Z}_8^\times \) and \( \mathbb{Z}_{10}^\times \).

49. Is \( \mathbb{Z}_{14}^\times \) cyclic?

50. Is \( \mathbb{Z}_{16}^\times \) cyclic?

   From the textbook: §1.3 #13 and §1.4 #28