At least two of these problems will be on Exam 4.

From the text: Section 3.4 #2, 4, 7, 9, 10 and Section 3.5 #2, 4, 6, 7, 10

**3.4.30** Let $\phi : \mathbf{R}^\times \to \mathbf{R}^\times$ be defined by $\phi(x) = x^3$, for all $x \in \mathbf{R}$. Show that $\phi$ is a group isomorphism.

**3.4.31** Let $G_1, G_2, H_1, H_2$ be groups, and suppose that $\theta_1 : G_1 \to H_1$ and $\theta_2 : G_2 \to H_2$ are group isomorphisms. Define $\phi : G_1 \times G_2 \to H_1 \times H_2$ by $\phi(x_1, x_2) = (\theta_1(x_1), \theta_2(x_2))$, for all $(x_1, x_2) \in G_1 \times G_2$. Prove that $\phi$ is a group isomorphism.

**3.4.33** Define $\phi : \mathbf{Z}_{30} \times \mathbf{Z}_2 \to \mathbf{Z}_{10} \times \mathbf{Z}_6$ by $\phi([n]_{30}, [m]_2) = ([n]_{10}, [4n + 3m]_6)$, First prove that $\phi$ is a well-defined function, and then prove that $\phi$ is a group isomorphism.

**3.4.34** Let $G$ be a group, and let $H$ be a subgroup of $G$. Prove that if $a$ is any element of $G$, then the subset

$$aHa^{-1} = \{g \in G \mid g = aha^{-1} \text{ for some } h \in H\}$$

is a subgroup of $G$ that is isomorphic to $H$.

**3.4.36** Let $G$ be an abelian group with subgroups $H$ and $K$. Prove that if $HK = G$ and $H \cap K = \{e\}$, then $G \cong H \times K$.

**3.5.21** Show that the three groups $\mathbf{Z}_6$, $\mathbf{Z}_9^\times$, and $\mathbf{Z}_{18}^\times$ are isomorphic to each other.

**3.5.22** Is $\mathbf{Z}_4 \times \mathbf{Z}_{10}$ isomorphic to $\mathbf{Z}_2 \times \mathbf{Z}_{20}$?

**3.5.23** Is $\mathbf{Z}_4 \times \mathbf{Z}_{15}$ isomorphic to $\mathbf{Z}_6 \times \mathbf{Z}_{10}$?

**3.5.28** Show that any cyclic group of even order has exactly one element of order 2.

**3.5.29** Use the the result in Problem 3.5.28 to show that the multiplicative groups $\mathbf{Z}_{15}^\times$ and $\mathbf{Z}_{21}^\times$ are not cyclic groups.