Hand in:

From the text:

Section §2.2 #3, 8

3. For each of the following relations on $\mathbb{R}$, determine which of the three conditions of Definition 2.2.1 hold.
   (a) For $a, b \in \mathbb{R}$, define $a \sim b$ if $a \leq b$.
   (b) For $a, b \in \mathbb{R}$, define $a \sim b$ if $a - b \in \mathbb{Q}$.
   (c) For $a, b \in \mathbb{R}$, define $a \sim b$ if $|a - b| \leq 1$.

8. For integers $m, n$, define $m \sim n$ if and only if $n \mid m^k$ and $m \mid n^j$ for some positive integers $k$ and $j$.
   (a) Show that $\sim$ is an equivalence relation on $\mathbb{Z}$.
   (b) Determine the equivalence classes $[1], [2], [6]$ and $[12]$.
   (c) Give a characterization of the equivalence class $[m]$.

From the Study Guide: page 22 §2.2 #23a,b; 24; 26; 28

23. For each of the following functions defined on the given set $S$, find $f(S)$ and $S/f$ and exhibit the one-to-one correspondence between them.
   (a) $f : \mathbb{Z}_{12} \to \mathbb{Z}_4 \times \mathbb{Z}_3$ defined by $f([x]_{12}) = ([x]_4, [x]_3)$ for all $[x]_{12} \in \mathbb{Z}_{12}$.
   (b) $f : \mathbb{Z}_{12} \to \mathbb{Z}_2 \times \mathbb{Z}_6$ defined by $f([x]_{12}) = ([x]_2, [x]_6)$ for all $[x]_{12} \in \mathbb{Z}_{12}$.

24. For $[x]_{15}$, let $f([x]_{15}) = [3x]_3^3$.
   (a) Show that $f$ defines a function from $\mathbb{Z}_{15}^\times$ to $\mathbb{Z}_5^\times$.
   (b) Find $f(\mathbb{Z}_{15}^\times)$ and $\mathbb{Z}_{15}^\times / f$ and exhibit the one-to-one correspondence between them.

26. For each of the following relations on the given set, determine which of the three conditions of Definition 2.2.1 hold.
   (a) For $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$, define $(x_1, x_2) \sim (y_1, y_2)$ if $2(x_1 - y_1) = 3(x_2 - y_2)$.
   (b) Let $P$ be the set of all people living in North America. For $p, q \in P$, define $p \sim q$ if $p$ and $q$ have the same biological mother.
   (c) Let $P$ be the set of all people living in North America. For $p, q \in P$, define $p \sim q$ if $p$ is the sister of $q$.

28. On $C[0, 1]$, define $f \sim g$ if $\int_0^1 f(x)dx = \int_0^1 g(x)dx$. Show that $\sim$ is an equivalence relation, and that the equivalence classes of $\sim$ are in one-to-one correspondence with the set of all real numbers.