1. (a) State the division algorithm.
   (b) State the Chinese remainder theorem.
   (c) Solve the following system of congruences:
   \[ x \equiv 13 \pmod{25} \quad x \equiv 9 \pmod{18} \]

2. (a) Use the Euclidean algorithm to find \([8]^{-1}_{27}\) (in \(\mathbb{Z}_{27}\)).
   (b) Find \(\varphi(27)\) and list all of its positive divisors.
   (c) Find the order of \([8]_{27}\) in the group \(\mathbb{Z}_{27}\).

3. Let \(\sigma = (3, 6, 8)(1, 9, 4, 3, 2, 7, 6, 8, 5)(2, 3, 9, 7) \in S_9\).
   (a) Write \(\sigma\) as a product of disjoint cycles.
   (b) Is \(\sigma\) an even permutation or an odd permutation?
   (c) What is the order of \(\sigma\) in \(S_9\)?
   (d) Compute \(\sigma^{-1}\) in \(S_9\).

4. (a) State the definition of an equivalence relation.
   (b) State the definition of a subgroup of a group.
   (c) Let \(G\) be a group, and let \(H\) be a subgroup of \(G\). For \(x, y \in G\), define \(x \sim y\) if \(x^{-1}y \in H\). Prove that \(\sim\) defines an equivalence relation on \(G\).

5. (a) State the definition of a one-to-one function.
   (b) State the definition of an onto function.
   (c) State the definition of an isomorphism of groups.
   (d) Let \(G_1, G_2\) be groups and let \(H_2\) be a subgroup of \(G_2\). Prove that if \(\phi : G_1 \to G_2\) is an isomorphism, then \(H_1 = \{g \in G_1 \mid \phi(g) \in H_2\}\) is a subgroup of \(G_1\).

6. (a) Let \(H\) and \(K\) be subgroups of the group \(G\). Prove that \(HK\) is a subgroup of \(G\) if and only if \(KH \subseteq HK\).
   (b) Let \(G = \mathbb{Z}_{10}^\times \times \mathbb{Z}_{10}^\times\), let \(H = \langle (3, 3) \rangle\) and let \(K = \langle (3, 7) \rangle\). List the elements of \(HK\).

7. (a) State the definition of a cyclic group.
   (b) Write out ONE of the following proofs from the text:
   I. Any subgroup of a cyclic group is cyclic.
   II. If \(G\) is a cyclic group of order \(n\), then \(G\) is isomorphic to \(\mathbb{Z}_n\).
8. (a) State the definition of the order of an element.

(b) Prove or disprove: If $a, b$ are elements of the group $G$ with $o(a) = m$ and $o(b) = n$, where $m, n$ are positive integers, then $o(ab) \leq \text{lcm}[m, n]$.

If you are unable to answer the general question, you can get partial credit for answering the question in the special case that $ab = ba$. 