Hand in:

From the Study Guide: pages 8–9, #39, 40

39. Find the quotient and remainder when \(a\) is divided by \(b\).
   (a) \(a = 12345, \ b = 100\)
   (b) \(a = -12345, \ b = 100\)
   (c) \(a = 123, \ b = 9\)
   (d) \(a = 12345, \ b = 9\)
   (e) \(a = 7862, \ b = 9\)
   (f) \(a = 123, \ b = 11\)
   (g) \(a = 12345, \ b = 11\)
   (h) \(a = 7862, \ b = 11\)

40. Find \(\gcd(252, 180)\) and write it as a linear combination of 252 and 180.

From the textbook, pages 14–15, #4(d) and 6(d), 7, 9, 13, 16, 21

4 (d) and 6 (d). Find \(\gcd(6540, 1206)\) and write it as a linear combination of 6540 and 1206.

7. Let \(a, b, c\) be integers. Give a proof for these facts about divisors:
   (a) If \(b\) \(\mid\) \(a\), then \(b\) \(\mid\) \(ac\).
   (b) If \(b\) \(\mid\) \(a\) and \(c\) \(\mid\) \(b\), then \(c\) \(\mid\) \(a\).
   (c) If \(c\) \(\mid\) \(a\) and \(c\) \(\mid\) \(b\), then \(c\) \(\mid\) \((ma + nb)\) for any integers \(m, n\).

9. Let \(a, b, c\) be integers.
   (a) Show that if \(b\) \(\mid\) \(a\) and \(b\) \(\mid\) \((a + c)\), then \(b\) \(\mid\) \(c\).
   (b) Show that if \(b\) \(\mid\) \(a\) and \(b\) \(\nmid\) \(c\), then \(b\) \(\nmid\) \((a + c)\).

13. Show that if \(n\) is any integer, then \((a + nb, b) = (a, b)\).

16. Let \(a, b, c\) be integers, with \(b > 0, c > 0\), and let \(q\) be the quotient and \(r\) the remainder when \(a\) is divided by \(b\).
   (a) Show that \(q\) is the quotient and \(rc\) is the remainder when \(ac\) is divided by \(bc\).
   (b) Show that if \(q'\) is the quotient when \(q\) is divided by \(c\), then \(q'\) is the quotient when \(a\) is divided by \(bc\). (Do not assume that the remainders are zero.)

21. Prove that the sum of the cubes of any three consecutive positive integers is divisible by 3.