Homework 2 due Tuesday, June 26, 2007

Hand in:

From the Study Guide: page 10, §1.2 #37, 40. 41, 45

37. Find the prime factorizations of 252 and 180 and use them to compute the greatest common divisor and least common multiple of 252 and 180.

40. Find the prime factorizations of 13651 and 3179 and use them to find $\gcd(13651, 3179)$.

41. Give a diagram of all divisors of 90, showing the divisibility relationships.

45. Let a, b, c be positive integers.
(a) Prove that if $\gcd(a, bc) = 1$ and $\gcd(b, c) = 1$, then $\gcd(ab, c) = 1$.
(b) Prove or disprove the following generalization of part (a): if $\gcd(b, c) = 1$, then $\gcd(a, bc) = \gcd(ab, c)$.

From the textbook, page 24, §1.2 #23, 25

23. Show that if n is a positive integer such that $2^n + 1$ is prime, then n is a power of 2.

25. If a, b, c are positive integers such that $a^2 + b^2 = c^2$, then (a, b, c) is called a **Pythagorean triple**. For example, $(3, 4, 5)$ and $(5, 12, 13)$ are Pythagorean triples. Assume that (a, b, c) is a Pythagorean triple in which the only common divisors of a, b, c are ± 1.
(a) Show that a and b cannot both be odd.
(b) Assume that a is even. Show that there exist relatively prime integers m and n such that $a = 2mn$, $b = m^2 - n^2$, and $c = m^2 + n^2$.
Hint: Factor $a^2 = c^2 - b^2$ after showing that $(c + b, c - b) = 2$.

From the Study Guide: page 13, §1.3 #45, 47

45. Solve the following congruences.
(a) $10x \equiv 5 \pmod{21}$
(b) $10x \equiv 5 \pmod{15}$
(c) $10x \equiv 4 \pmod{15}$
(d) $10x \equiv 4 \pmod{14}$

47. Solve the following congruence. $20x \equiv 12 \pmod{72}$