

Homework 3

due Wednesday, February 3, 2010, in class

Hand in:

From the Study Guide: page 13, §1.3 #48, 49, 50, 54, 55

48. Solve the following congruence. $25x \equiv 45 \pmod{60}$ Note: The smallest positive solution of the congruence $ax \equiv 0 \pmod{n}$ is called the *additive order* of a modulo n .

49. Find the additive orders of each of the following elements, by solving the appropriate congruences.

(a) 4, 5, 6 modulo 24

(b) 4, 5, 6 modulo 25

50. Find the additive orders of each of the following elements, by solving the appropriate congruences.

(a) 7, 8, 9 modulo 24

(b) 7, 8, 9 modulo 25

54. Solve the following system of congruences:

$$x \equiv 13 \pmod{25} \quad x \equiv 9 \pmod{18}$$

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$$x \equiv 9 \pmod{25} \quad x \equiv 13 \pmod{18}$$

From the text:

Section §1.3 #10, 13, 24, 27

10. Let a, b, n be positive integers. Prove that if $a \equiv b \pmod{n}$, then $(a, n) = (b, n)$.

13. Prove that the sum of the cubes of any three consecutive positive integers is divisible by 9.

24. Show that the remainder of an integer n when divided by 9 is the same as the remainder of the sum of its digits when divided by 9.27. Prove that in any Pythagorean triple (a, b, c) , either a or b is divisible by 3, and one of a, b, c is divisible by 5.