

Homework 5

due Monday, July 6, at 5:00 pm

Recommended: §2.1 #1 – 10

Hand in, from the text:

Section §2.1 #9 b,d; 11; 12; 18 – 20

9. (b) Show that $g : \mathbf{Z}_8 \rightarrow \mathbf{Z}_{12}$ defined by $g([x]_8) = [6x]_{12}$ is a well-defined function.

(d) Show that $p : \mathbf{Z}_{10} \rightarrow \mathbf{Z}_5$ defined by $p([x]_{10}) = [x^2 + 2x - 1]_5$ is a well-defined function.

11. Let k and n be positive integers. For a fixed $m \in \mathbf{Z}$, define the formula $f : \mathbf{Z}_n \rightarrow \mathbf{Z}_k$ by $f([x]_n) = [mx]_k$, for $x \in \mathbf{Z}$. Show that f defines a function if and only if $k|mn$.

12. Let k, m, n be positive integers such that $k|mn$. Show that the function $f : \mathbf{Z}_n \rightarrow \mathbf{Z}_k$ defined in Exercise 11 by $f([x]_n) = [mx]_k$ is a one-to-one correspondence if and only if $k = n$ and $(m, n) = 1$.

18. Let A be a nonempty set, and let $f : A \rightarrow B$ be a function. Prove that f is one-to-one if and only if there exists a function $g : B \rightarrow A$ such that $g \circ f = 1_A$.

19. Let $f : A \rightarrow B$ be a function. Prove that f is one-to-one if and only if $f \circ h = f \circ k$ implies $h = k$, for every set C and all choices of functions $h : C \rightarrow A$ and $k : C \rightarrow A$.

20. Define $f : \mathbf{Z}_{mn} \rightarrow \mathbf{Z}_m \times \mathbf{Z}_n$ by $f([x]_{mn}) = ([x]_m, [x]_n)$. Show that f is a function and that f is onto if and only if $\gcd(m, n) = 1$.