Homework 5 due Wednesday, February 24, 2010, in class

Hand in:

From the Study Guide: page 19, §2.1 #37, 39, 41, 43, 45 (2 pts each)

37. In each of the following parts, determine whether the given function is one-to-one and whether it is onto. If the function is one-to-one and onto, then find its inverse.
(a) $f : \mathbf{Z}_{12} \to \mathbf{Z}_{12}$; $f([x]_{12}) = [7x + 3]_{12}$, for all $[x]_{12} \in \mathbf{Z}_{12}$
(b) $f : \mathbf{Z}_{12} \to \mathbf{Z}_{12}$; $f([x]_{12}) = [8x + 3]_{12}$, for all $[x]_{12} \in \mathbf{Z}_{12}$
(c) $f : \mathbf{Z}_{12} \to \mathbf{Z}_{12}$; $f([x]_{12}) = [x]^2_{12}$, for all $[x]_{12} \in \mathbf{Z}_{12}$
(d) $f : \mathbf{Z}_{12} \to \mathbf{Z}_{12}$; $f([x]_{12}) = [x]^3_{12}$, for all $[x]_{12} \in \mathbf{Z}_{12}$
(e) $f : \mathbf{Z}^\times_{12} \to \mathbf{Z}^\times_{12}$; $f([x]_{12}) = [x]^2_{12}$, for all $[x]_{12} \in \mathbf{Z}^\times_{12}$
(f) $f : \mathbf{Z}^\times_{12} \to \mathbf{Z}^\times_{12}$; $f([x]_{12}) = [x]^3_{12}$, for all $[x]_{12} \in \mathbf{Z}^\times_{12}$

39. Define $f : \mathbf{Z}_{10} \to \mathbf{Z}^\times_{11}$ by $f([m]_{10}) = [2]^m_{11}$, for all $[m]_{10} \in \mathbf{Z}_{10}$.
(a) Show that f is a well-defined function.
(b) Is f one-to-one and onto?

41. Show that each of the following formulas yields a well-defined function.
(a) $f : \mathbf{Z}_8 \to \mathbf{Z}_8$ defined by $f([x]_8) = [3x^2 - 3x + 1]_8$, for all $[x]_8 \in \mathbf{Z}_8$
(b) $f : \mathbf{Z}_{12} \to \mathbf{Z}_8$ defined by $f([x]_{12}) = [2x^2 - 4x + 6]_8$, for all $[x]_{12} \in \mathbf{Z}_{12}$
(b) $f : \mathbf{Z}^\times_{15} \to \mathbf{Z}^\times_5$ defined by $f([x]_{15}) = [3x^3]_5$, for all $[x]_{15} \in \mathbf{Z}^\times_{15}$

43. Let n be a positive integer. Show that $i : \mathbf{Z}^\times_n \to \mathbf{Z}^\times_n$ defined by $i([x]_n) = [x]^{-1}_n$, for all $[x]_n \in \mathbf{Z}^\times_n$ is a well-defined one-to-one correspondence.

45. Let m, n be positive integers with $m \mid n$, and let k be any integer. Show that $f : \mathbf{Z}^\times_n \to \mathbf{Z}^\times_m$ defined by $f([x]_n) = [x]^k_m$, for all $[x]_n \in \mathbf{Z}^\times_n$ is a well-defined function.

From the text:

Section §2.1 #11, 15, 19, 20

11. Let k and n be positive integers. For a fixed $m \in \mathbf{Z}$, define the formula $f : \mathbf{Z}_n \to \mathbf{Z}_k$ by $f([x]_n) = [mx]_k$, for $x \in \mathbf{Z}$. Show that f defines a function if and only if $k|mn$.

15. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is one-to-one, then f is one-to-one, and that if $g \circ f$ is onto, then g is onto.

19. Let $f : A \to B$ be a function. Prove that f is one-to-one if and only if $f \circ h = f \circ k$ implies $h = k$, for every set C and all choices of functions $h : C \to A$ and $k : C \to A$.

20. Define $f : \mathbf{Z}_{mn} \to \mathbf{Z}_m \times \mathbf{Z}_n$ by $f([x]_{mn}) = ([x]_m, [x]_n)$. Show that f is a function and that f is onto if and only if $\gcd(m, n) = 1$.