

Homework 6

due Wednesday, July 8, at 5:00 pm

Hand in:

From the text:

Section §2.2 #3, 8

3. For each of the following relations on \mathbf{R} , determine which of the three conditions of Definition 2.2.1 hold.
- For $a, b \in \mathbf{R}$, define $a \sim b$ if $a \leq b$.
 - For $a, b \in \mathbf{R}$, define $a \sim b$ if $a - b \in \mathbf{Q}$.
 - For $a, b \in \mathbf{R}$, define $a \sim b$ if $|a - b| \leq 1$.
8. For integers m, n , define $m \sim n$ if and only if $n | m^k$ and $m | n^j$ for some positive integers k and j .
- Show that \sim is an equivalence relation on \mathbf{Z} .
 - Determine the equivalence classes $[1]$, $[2]$, $[6]$ and $[12]$.
 - Give a characterization of the equivalence class $[m]$.

From the Study Guide: page 22 §2.2 #23a,b; 24; 26; 28

23. For each of the following functions defined on the given set S , find $f(S)$ and S/f and exhibit the one-to-one correspondence between them.
- $f : \mathbf{Z}_{12} \rightarrow \mathbf{Z}_4 \times \mathbf{Z}_3$ defined by $f([x]_{12}) = ([x]_4, [x]_3)$ for all $[x]_{12} \in \mathbf{Z}_{12}$.
 - $f : \mathbf{Z}_{12} \rightarrow \mathbf{Z}_2 \times \mathbf{Z}_6$ defined by $f([x]_{12}) = ([x]_2, [x]_6)$ for all $[x]_{12} \in \mathbf{Z}_{12}$.
24. For $[x]_{15}$, let $f([x]_{15}) = [3x]_5^3$.
- Show that f defines a function from \mathbf{Z}_{15}^\times to \mathbf{Z}_5^\times .
 - Find $f(\mathbf{Z}_{15}^\times)$ and \mathbf{Z}_{15}^\times/f and exhibit the one-to-one correspondence between them.
26. For each of the following relations on the given set, determine which of the three conditions of Definition 2.2.1 hold.
- For $(x_1, x_2), (y_1, y_2) \in \mathbf{R}^2$, define $(x_1, x_2) \sim (y_1, y_2)$ if $2(x_1 - y_1) = 3(x_2 - y_2)$.
 - Let P be the set of all people living in North America. For $p, q \in P$, define $p \sim q$ if p and q have the same biological mother.
 - Let P be the set of all people living in North America. For $p, q \in P$, define $p \sim q$ if p is the sister of q .
28. On $\mathcal{C}[0, 1]$, define $f \sim g$ if $\int_0^1 f(x)dx = \int_0^1 g(x)dx$. Show that \sim is an equivalence relation, and that the equivalence classes of \sim are in one-to-one correspondence with the set of all real numbers.