

Homework 6

due Monday, March 1, 2010, in class

Hand in:

From the text:

Section §2.2 #1, 3, 5, 7, 10 (5 pts each)

1. For each of the following functions, find $f(S)$ and S/f and exhibit the one-to-one correspondence between them.
 - (a) $f : \mathbf{Z} \rightarrow \mathbf{C}$ given by $f(n) = i^n$ for all $n \in \mathbf{Z}$
 - (b) $g : \mathbf{Z} \rightarrow \mathbf{Z}_{12}$ given by $g(n) = [8n]_{12}$ for all $n \in \mathbf{Z}$
 - (c) $h : \mathbf{Z}_{12} \rightarrow \mathbf{Z}_{12}$ defined by $h([x]_{12}) = [9x]_{12}$
 - (d) $k : \mathbf{Z}_{12} \rightarrow \mathbf{Z}_{12}$ defined by $k([x]_{12}) = [5x]_{12}$

3. For each of the following relations on \mathbf{R} , determine which of the three conditions of Definition 2.2.1 hold.
 - (a) For $a, b \in \mathbf{R}$, define $a \sim b$ if $a \leq b$.
 - (b) For $a, b \in \mathbf{R}$, define $a \sim b$ if $a - b \in \mathbf{Q}$.
 - (c) For $a, b \in \mathbf{R}$, define $a \sim b$ if $|a - b| \leq 1$.

5. On \mathbf{R}^2 , define $(a_1, a_2) \sim (b_1, b_2)$ if $a_1^2 + a_2^2 = b_1^2 + b_2^2$. Check that this defines an equivalence relation. What are the equivalence classes?

7. Define an equivalence relation on the set \mathbf{R} that partitions the real line into subsets of length 1.

10. Let S be a set and let $2^S = \{A \mid A \subseteq S\}$ be the collection of all subsets of S . Define \sim on 2^S by letting $A \sim B$ if and only if there exists a one-to-one correspondence from A to B .
 - (a) Show that \sim is an equivalence relation on 2^S .
 - (b) If $S = \{1, 2, 3, 4\}$, list the elements of 2^S and find each equivalence class determined by \sim .