MATH 421 TEST I (TAKE-HOME RETEST) March 5, 2004

Prof. John Beachy Show all of the work necessary to justify your answers.

If you choose to hand in these problems, I will replace your score on problems 3 and 4 of Test I with the average of that score and your score on problems 6, 7, and 8.

6. (15 pts) Let $G = \mathbf{Z}_{20}^{\times}$ (the multiplicative group of nonzero congruence classes modulo 20), and let N be the subgroup $\{1, -1\} = \langle -1 \rangle$.

   (a) Find the cosets of N.

   (b) Find the order of each coset of N in the factor group G/N.

   (c) Is G/N a cyclic group?

7. (15 pts) Let G be the group $\mathbf{Z}_3 \times \mathbf{Z}_6$, and let N be the subgroup $\langle ([2]_3, [2]_6) \rangle$ generated by the element $([2]_3, [2]_6)$.

   (a) Find the cosets of N.

   (b) Find the order of each coset of N in the factor group G/N.

   (c) Is G/N a cyclic group?

8. (15 pts) Assume that the dihedral group of order $2n$ is given as all elements of the form

$$D_n = \{a^i b^j \mid 0 \le i < n,\ 0 \le j < 2\} ,$$

   subject to the conditions that $o(a) = n$, $o(b) = 2$, and $ba = a^{-1}b$. Let N be any subgroup of D_n with $N \subseteq \langle a \rangle$, where $\langle a \rangle$ is the cyclic subgroup generated by a.

   (a) Show that N is a normal subgroup of D_n.

   (b) Show that if $n > 2$ and a^2 is not in N, then the factor group D_n/N is not abelian. (You may use the homework problem which states that a factor group G/N is abelian if and only if $xyx^{-1}y^{-1} \in N$, for all $x, y \in G$.)