1. Define $T, U : \mathbf{R}_3 \to \mathbf{R}_3$ by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + x_2, x_2 + x_3)$ and $U(x_1, x_2, x_3) = (x_1 - x_3, x_1 + x_2, -x_1 - 3x_2 + x_3)$.
   (a) Show that T is not one-to-one. (see p87 #6e)
   (b) Compute TU and UT. Are they equal?
   (c) Find the null space of UT and show that it contains the null space of T.

2. Let $T : V \to W$ be a linear transformation.
   (a) Show that if V_1 is a subspace of V, then $T(V_1) = \{w \in W \mid w = T(v) \text{ for some } v \in V\}$ is a subspace of W. (see p87, #5)
   (b) Show that if W_1 is a subspace of W, then $T^{-1}(W_1) = \{v \in V \mid T(v) \in W_1\}$ is a subspace of V.

3. Curtis, p88, #10

4. Curtis, p98, #5

5. Prove that the only 3×3 matrices which commute with all other 3×3 matrices are the scalar matrices. (A diagonal matrix is called a *scalar* matrix if all of the entries are the same.)

6. Curtis, p99, #7c

7. Curtis, p99, #7e

8. Curtis, p108, #8

9. Curtis, p108, #11

10. Let V be a vector space, and let $T \in L(V, V)$. Prove that if $T^2 = T$, then $V = n(T) \oplus T(V)$.