MATH 423 TEST I 7/3/2006

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Each question is worth 20 points. The field of real numbers will be denoted by $\mathbf{R}$.

1. Give careful statements of the following definitions:

   (a) *linear dependence;*

   (b) *generating set* (or *spanning set*);

   (c) *basis;*

   (d) *linear manifold in* $\mathbf{R}_n$.

2. Prove Lemma 7.1 from the text: Let $\{a_1, \ldots, a_m\}$ be a linearly dependent subset of the vector space V. If the set $\{a_1, \ldots, a_{m-1}\}$ is linearly independent, then a_m is a linear combination of $a_1, \ldots, a_{m-1}$.

3. (a) Find a basis for the subspace S of $\mathbf{R}_4$ generated by the vectors $\langle 3, -1, 1, 2 \rangle$, $\langle 4, -1, -2, 3 \rangle$, $\langle 10, -3, 0, 7 \rangle$, $\langle -1, 1, -7, 0 \rangle$.

   (b) Find a set of homogeneous equations whose solution space is S.

   (c) Find a system of nonhomogeneous equations whose set of solutions is the linear manifold with directing space S and which passes through $\langle 1, 1, 1, 1 \rangle$.

4. Let V be the vector space over $\mathbf{R}$ consisting of all 3×3 matrices with entries in $\mathbf{R}$. Let W_1 be the set of all symmetric matrices in V (that is, all 3×3 matrices A with ${}^tA = A$). Let W_2 be the set of all skew symmetric matrices in V (that is, all 3×3 matrices A with ${}^tA = -A$).

   (a) Show that W_1 is a subspace of V, and find its dimension.

   (b) Show that W_2 is a subspace of V, and find its dimension.

   (c) Show that $V = W_1 \oplus W_2$.

   Recall that a vector space V is the *direct sum* of subspaces W_1 and W_2, denoted by $V = W_1 \oplus W_2$, if $W_1 + W_2 = V$ and $W_1 \cap W_2 = \{0\}$.

5. Let M_1 and M_2 be linear manifolds in $\mathbf{R}_n$. Prove that if $M_1 \cap M_2 \neq \emptyset$, then $M_1 \cap M_2$ is a linear manifold in $\mathbf{R}_n$.