MATH 423 TEST IV 8/10/2006

Professor John Beachy

Choose 3 of the first 4 problems, for 20 points each.
Work problem 5, for 40 points (the parts can be done independently).

The underlying assumption is that V is a finite dimensional vector space.

Choose 3 **of the following** 4 **problems** (60 pts). I recommend 1–3.

1. Let $T \in L(V, V)$.

(a) Define: *T-invariant subspace; generalized eigenspace of T.*

(b) Prove that each generalized eigenspace of T is a T-invariant subspace of V.

2. Let A be a 10×10 matrix with real entries, whose characteristic polynomial is $(x-3)^{10}$. Suppose that the nullspace $n(A-3I)$ has dimension 5, that $n((A-3I)^4)$ has dimension 10, and that $n((A-3I)^3)$ has dimension less than 10.

(a) In the Jordan canonical form for A, how many secondary blocks will there be? Explain what a secondary block is, and explain how you got your answer.

(b) List all of the possible Jordan forms for A. Give a brief explanation for your answer.

3. Let $D = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ be the matrix that represents differentiation (from polynomials of degree ≤ 3 to polynomials of degree ≤ 3). Find the matrix which gives the Jordan canonical form for D. Give an explanation for your answer.

4. Let $W_1 \subseteq W_2$ be subspaces of V. Give the definition of a complementary basis for W_1 in W_2. Then prove the following lemma from the class notes:

Let $L : V \to V$ be a linear transformation. Let $\{\mathbf{u}_1, \ldots, \mathbf{u}_k\}$ be a complementary basis for $n(L^m)$ in $n(L^{m+1})$. Then $\{L(\mathbf{u}_1), \ldots, L(\mathbf{u}_k)\}$ is part of a complementary basis for $n(L^{m-1})$ in $n(L^m)$.

5. (40 pts) Let $A = \begin{bmatrix} 0 & -8 & 1 & 2 \\ 0 & 2 & 0 & 0 \\ -1 & -3 & 2 & 1 \\ -2 & -8 & 1 & 4 \end{bmatrix}$

(a) Find the characteristic polynomial of A by calculating $D(xI - A)$. (Answer: $(x-2)^4$)

(b) Find the dimension of $n(A - 2I)$. (Answer: 2)

(c) Find the dimension of $n((A - 2I)^2)$. (Answer: 3)

(d) Give the Jordan canonical form for A, and explain your answer.