Curtis, p26, #10: Show that the midpoints of any quadrilateral are the vertices of a parallelogram.

Hint: Use the midpoint formula on p24, then subtract one midpoint from the other to obtain the vector joining them. Compare opposite sides.

Curtis, p32, #1: Determine which of the following subsets of $\mathbf{R}_n$ are subspaces:

(a) $\{(a_1, \ldots, a_n) \mid a_1 = 1\}$
(b) $\{(a_1, \ldots, a_n) \mid a_1 = 0\}$
(c) $\{(a_1, \ldots, a_n) \mid a_1 + 2a_2 = 0\}$
(d) $\{(a_1, \ldots, a_n) \mid a_1 + a_2 + \ldots + a_n = 1\}$
(e) $\{(a_1, \ldots, a_n) \mid A_1a_1 + A_2a_2 + \ldots + A_na_n = 0\}$, for fixed $A_1, \ldots, A_n$ in $\mathbf{R}$
(e) $\{(a_1, \ldots, a_n) \mid A_1a_1 + A_2a_2 + \ldots + A_na_n = B\}$, for fixed $A_1, \ldots, A_n, B$ in $\mathbf{R}$
(f) $\{(a_1, \ldots, a_n) \mid a_1^2 = a_2\}$

Hint: Check that the zero vector belongs, and that the set is closed under addition and scalar multiplication.

Curtis, p33, #2: Let $\mathcal{F}(\mathbf{R})$ be the vector space of all real-valued functions defined on the set $\mathbf{R}$ of real numbers. (see p19) Show that each of the following subsets is a subspace.

The set $C(\mathbf{R})$ of continuous functions in $\mathcal{F}(\mathbf{R})$.
The set $D(\mathbf{R})$ of differentiable functions in $\mathcal{F}(\mathbf{R})$.
The set $P(\mathbf{R})$ of polynomial functions in $\mathcal{F}(\mathbf{R})$.

Curtis, p33, #3: Determine which of the following subsets of $C(\mathbf{R})$ are subspaces of $C(\mathbf{R})$.

(a) $\{f \in C(\mathbf{R}) \mid f \in P(\mathbf{R})\}$
(b) $\{f \in C(\mathbf{R}) \mid f(1/2) \text{ is rational}\}$
(c) $\{f \in C(\mathbf{R}) \mid f(1/2) = 0\}$
(d) $\{f \in C(\mathbf{R}) \mid \int_0^1 f(t)dt = 1\}$
(e) $\{f \in C(\mathbf{R}) \mid \int_0^1 f(t)dt = 0\}$
(f) $\{f \in C(\mathbf{R}) \mid f'(t) = 0\}$
(g) $\{f \in C(\mathbf{R}) \mid af''(t) + bf'(t) + cf = 0\}$ for some $a, b, c \in \mathbf{R}$
(h) $\{f \in C(\mathbf{R}) \mid af''(t) + bf'(t) + cf = g\}$ for some $a, b, c \in \mathbf{R}$ and some fixed $g \in C(\mathbf{R})$