Math 423 | Homework 7 | due in class on Tuesday, 8/8/06

1. Curtis, page 201, #3
2. Curtis, page 201, #4
3. Curtis, page 201, #6
4. Curtis, page 215, #4 (Important: I changed to #4 instead of #5 because I wanted to do #5 in the class notes.)
5. Curtis, page 140, #3
6. Let $f(x) = x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 \in F[x]$. The *companion matrix* of $f(x)$ is the matrix

   $$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & 0 & \cdots & 0 & -a_1 \\ 0 & 1 & 0 & \cdots & 0 & -a_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -a_{n-2} \\ 0 & 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix}.$$

   Show that the characteristic polynomial of A is $f(x)$, and thus $f(A) = 0$.

   Hint: Calculate the determinant by expanding by cofactors along the last column. Note that there is a bit of a discussion on the companion matrix on page 221 of Curtis.

Math 423 | Homework 8 | due in class on Wednesday, 8/9/06

1. Over **R**, determine which of these matrices are similar by finding their Jordan canonical form. First show that for each matrix the characteristic polynomial is $(x-2)^2(x-1)$.

   $$A = \begin{bmatrix} -3 & 3 & -2 \\ -7 & 6 & -3 \\ 1 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & -1 \\ -4 & 4 & -2 \\ -2 & 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & -1 & -1 \\ -3 & -1 & -2 \\ 7 & 5 & 6 \end{bmatrix}$$

2. Curtis, page 226, #6
3. Find all possible Jordan canonical forms of each of the following matrices, over the field of complex numbers.

   (a) The matrix A has characteristic polynomial $(x-2)^4(x-10)^3$ and minimal polynomial $(x-2)^3(x-10)$.

   (b) The matrix B has rank 4, characteristic polynomial x^7, and minimal polynomial x^3.
4. Let A be a matrix over the field of real numbers. If the characteristic polynomial of A is $(x-2)(x-3)^6$ and the minimal polynomial of A is $(x-2)(x-3)^3$, list the possible Jordan canonical forms for A.