MATH 520 Midterm Exam 10/6/2004
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The first examination is scheduled for Friday, October 15. It will cover sections 1.1–1.4 and 2.1–2.7. Since sections 1.1 and 1.2 are really a review, you should read them, but don't expect specific questions on that material. Sections 1.3 and 1.4 are more important: the notion of a semidirect product is new, as are the first and second isomorphism theorems. You should review carefully, because the concepts of cosets and factor groups are so important.

In Chapter 2, we have focused on the Sylow theorems, the structure of finite abelian groups, and we will cover the concepts of a solvable group, and will prove that A_n is simple, for $n \geq 5$. This is in preparation for studying Galois theory, where we will prove that there exist polynomial equations of degree 5 that cannot be solved by radicals. The proof is given by showing that each polynomial has an associated group, called its Galois group. Then the polynomial equation is solvable by radicals if and only if its Galois group is a solvable group. The symmetric group S_5 is not solvable, so after doing the general theory we only need to produce a polynomial whose Galois group is S_5. (It took about 300 years for mathematicians to answer this question.)

There is some material in the text that I don't consider crucial, so because of time pressure we will not cover it in class, and I will not ask questions on it (for your own information, it wouldn't hurt to read the statements of the results). Here are the pages to omit: Section 2.5, pp. 88–90, Lemma 2.5.9–Corollary 2.5.13; Section 2.7, pp. 101–103, Lemma 2.7.7–Theorem 2.7.9. The first omission deals with the question of when $\mathbf{Z}_n^\times$ is cyclic, and the second with the question of when the projective special linear group over a finite field is simple. I'm sorry that we don't have time to classify all groups of order < 16 (this is done in Section 2.9) since part of the reason for studying semidirect products is to be able to describe a number of groups as semidirect products of cyclic groups of small order. I should also note that Section 2.8 studies groups that are a direct product of their Sylow subgroups (generalizing finite abelian groups).