Math 523 | Homework 4 | Due 7/21/2008

Total: 40 points

From Section 2 of the lecture notes:

In these exercises, let F be a field, let $p(x)$ be a nonconstant polynomial in $F[x]$, and let $R = F[x]/\langle p(x)\rangle$.

1. (5 pts) If $f(x)$ is any polynomial, show that $\langle [f(x)]\rangle = \langle [d(x)]\rangle$ in R, where $d(x) = \gcd(f(x), p(x))$.

2. (5 pts) Suppose that $p(x) = g(x)h(x)$, where $\gcd(g(x), h(x)) = 1$. Show that in R we have

$$\langle [g(x)]\rangle = \{[f(x)] \in R \mid f(x)h(x) \equiv 0 \pmod{p(x)}\} .$$

4. (5 pts) In $Z_2[x]/\left\langle x^{15} - 1\right\rangle$, find the idempotent generator for the ideal $\left\langle x^4 + x + 1\right\rangle$. Be sure to show that your answer is in fact idempotent.

From Section 3 of the lecture notes:

1. (15 pts) Find the splitting fields over $\mathbf{Z}_2$ for the following polynomials:
(a) $x^2 + x + 1$
(b) $x^2 + 1$
(c) $x^3 + x + 1$
(d) $x^3 + x^2 + 1$

2. (5 pts) Find the splitting field for $x^p - x$ over $\mathbf{Z}_p$.

3. (5 pts) Show that if F is an extension field of K of degree 2, then F is the splitting field over K for some polynomial.