Math 523 | Homework 5 | Due 7/28/2008

Total: 20 points

From Section 4 of the lecture notes:

1. Give a multiplication table for $GF(3^2)$. Find all generators for the cyclic group $GF(3^2)^\times$, and find the minimal polynomial of each generator over $\mathbf{Z}_3$.
2. Find all generators for the cyclic group of nonzero elements of $GF(2^4)$, and find the minimal polynomial of each generator over $\mathbf{Z}_2$.
3. Let m, n be positive integers with $\gcd(m, n) = d$. Show that over any field the greatest common divisor of $x^m - 1$ and $x^n - 1$ is $x^d - 1$.

   Hint: Use the Euclidean algorithm.
4. If E and F are subfields of $GF(p^k)$ with p^m and p^n elements respectively, use the previous exercise to show that $E \cap F$ contain p^d elements, where $d = \gcd(m, n)$.