Chapter 1

Equations and Inequalities

Section summaries

Section 1.1 Linear Equations

A **linear equation** has the form $ax + b = 0$, where $a \neq 0$. It is solved by shifting b to the other side of the equation and dividing by a, to get $x = -\frac{b}{a}$.

Review problems: p95 #39,55,61,89

Section 1.2 Quadratic Equations

A **quadratic equation** has the form $ax^2 + bx + c = 0$, where $a \neq 0$. To solve a quadratic equation, first try to factor it and set each factor equal to zero. If you can't see factors right away, then use the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \,.$$

You *must* know the quadratic formula. If $b^2 - 4ac$ is negative, then we have the square root of a negative number, so in this case the equation has no solution (in the set of real numbers).

The quadratic formula is derived using the technique of **completing the square**. This technique will also be used in later sections, so you need to know how to use it. The technique is described on page 100; it is based on the fact that if you are given $x^2 + mx$, then adding $\left(\frac{m}{2}\right)^2$ gives $x^2 + mx + \left(\frac{m}{2}\right)^2$, which is equal to $\left(x + \frac{m}{2}\right)^2$.

Review problems: p107 #43,67,89,105,113

Section 1.4 Radical Equations

Radical equations are equations involving a square root, cube root, etc. Solving them involves raising each side of the equation to some power. Example 3 on page 119 provides a good illustration. In solving $\sqrt{2x+3} - \sqrt{x+2} = 2$ it might be tempting to square both sides, but the work is easier if you first solve for $\sqrt{2x+3}$, as shown in the example. Sometimes an equation may be a"disguised" quadratic equation. Making a substitution can be the easiest way to see the it looks like a quadratic. Then you solve it like you did in Section 1.2. The last step is to change back to the original variable. See Example 6 on page 121 for a good example of the technique.

Review problems: p122 #15,29,43,57,75

Section 1.5 Solving Inequalities

Read page 128 carefully. In solving an inequality, only a few things are different from solving an equation. If you multiply or divide both sides by a *negative* number, you must remember to reverse the direction of the inequality.

Review problems: p132 #59,77,83,91,107

Section 1.6 Equations and Inequalities Involving Absolute Value

The absolute value notation $|\quad|$ was introduced to find the distance between two points on the number line (review Example 3 on page 19).

An inequality like $|x| < a$ can be written as $|x - 0| < a$, and can be interpreted as saying that the distance between x and 0 is less than a. The solution on the number line gives the interval $-a < x < a$.

On the other hand, $a < |x|$ says that the distance between x and 0 is greater than a. On the number line, this translates into two intervals on either side of 0. Any value of x with $a < x$ will satisfy the inequality, but so will any value of x with $x < -a$.

Summary: $|u| < a$ can be simplified to the double inequality $-a < u < a$ while $|u| > a$ can be simplified to two inequalities: $u > a$ OR $u < -a$. (See page 136 and page 137.)

Review problems: p138 #41,45,47,63,73

Section 1.7 Problem Solving

For problems involving motion, use distance = rate · time. In some other types of problems, the principle is the same: rate · time gives you the total amount produced. This works for problems involving jobs.

It's hard to give general methods for such a variety of problems; use your common sense, and practice a lot of them.

Review problems: p146 #19,25,31,35,37,49

Sample Questions

1.1 A. Solve for x: $7 - 2x = 9 + 3x$

(a) $x = 2$
(b) $x = -2$
(c) $x = \frac{2}{5}$
(d) $x = -\frac{2}{5}$
(e) $x = -3$

1.1 B. Solve the equation: $1 - \frac{1}{2}x = 6 + x$.

(a) $x = \frac{10}{3}$
(b) $x = -\frac{10}{3}$
(c) $x = 2$
(d) $x = -3$
(e) None of these

1.1 Example 6. Solve the equation: $\dfrac{3x}{x-1} + 2 = \dfrac{3}{x-1}$

(a) $x = 1$
(b) $x = 5$
(c) $x = \frac{1}{5}$
(d) There is no solution
(e) None of these

1.1 #51. Solve this equation: $\dfrac{2x}{x^2-4} = \dfrac{4}{x^2-4} - \dfrac{3}{x+2}$

(a) $x = 2$ or $x = -2$
(b) $x = -1$
(c) $x = 2$
(d) There is no solution
(e) None of these

1.1 #59. Solve this equation: $\dfrac{4}{x-2} = \dfrac{-3}{x+5} + \dfrac{7}{x^2+3x-10}$

(a) $x = 2$
(b) $x = 1$
(c) $x = -\frac{19}{17}$
(d) There is no solution
(e) None of these

1.1 C. Going into the final exam, which will count as two tests, Brooke has test scores of 80, 83, 71, 61, and 89. What score does Brooke need on the final in order to have an average score of 80?

(a) 90
(b) 88
(c) 85
(d) 82
(e) None of these

1.1 #96. A wool suit, discounted by 30% for a clearance sale, has a price tag of $399. What was the suit's original price?

(a) Not enough information to determine
(b) $306.92 (approximately)
(c) $518.70
(d) $532
(e) $570

1.2 A.Solve for x: $x^2 - 3x + 2 = 0$

(a) $x = 1$ or $x = 2$
(b) $x = 1$ or $x = -2$
(c) $x = 2$ or $x = 3$
(d) $x = 2$ or $x = -3$
(e) None of these

1.2 B. Solve for x: $x^2 - 2x = 4$

(a) $x = 4$ or $x = 2$

(b) $x = \dfrac{4+\sqrt{5}}{2}$ or $x = \dfrac{2-\sqrt{5}}{2}$

(c) $x = \dfrac{2+\sqrt{5}}{2}$ or $x = \dfrac{4-\sqrt{5}}{2}$

(d) $x = 1 + \sqrt{5}$ or $x = 1 - \sqrt{5}$

(e) None of these

1.2 C. Find the value of a so that $x^2 + ax + \frac{1}{9}$ is a perfect square.

(a) $a = \frac{1}{3}$ (b) $a = \frac{2}{3}$ (c) $a = \frac{1}{9}$ (d) $a = \frac{2}{9}$ (e) None of these

1.2 D. Find the value of k so that $x^2 - \frac{3}{2}x + k$ is a perfect square.

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{9}{16}$ (d) $-\frac{9}{16}$ (e) None of these

1.2 #51. Use the quadratic formula to solve this equation: $\frac{2}{3}x^2 - \frac{5}{3}x + 1 = 0$

(a) $x = -1$ and $x = -2$

(b) $x = -1$ and $x = -\frac{3}{2}$

(c) $x = 1$ and $x = -\frac{3}{2}$

(d) $x = 1$ and $x = 2$

(e) None of these

1.2 #67. Find the real solutions, if any: $4 - \dfrac{1}{x} - \dfrac{2}{x^2} = 0$

(a) $x = \dfrac{-1 \pm \sqrt{17}}{8}$

(b) $x = \dfrac{1 \pm \sqrt{33}}{8}$

(c) $x = \dfrac{1}{2}$ or $x = -\dfrac{1}{4}$

(d) There is no solution

(e) None of these

1.2 #87. Use the quadratic formula to find the real solutions: $x^2 + \sqrt{2}x = \frac{1}{2}$

(a) $x = \dfrac{\sqrt{2} \pm 2}{2}$

(b) $x = \dfrac{-\sqrt{2} \pm 2}{2}$

(c) $x = \dfrac{\sqrt{2} \pm \sqrt{3}}{2}$

(d) $x = \dfrac{-\sqrt{2} \pm \sqrt{3}}{2}$

(e) None of these

1.2 #105. An open box is to be constructed from a square piece of sheet metal by removing a square of side 1 foot from each corner and turning up the edges. If the box is to hold 4 cubic feet, then the dimensions of the sheet metal should be

(a) 1 foot by 1 foot

(b) 2 feet by 2 feet

(c) 4 feet by 4 feet

(d) 8 feet by 8 feet

(e) None of these

1.2 #107. A ball is thrown vertically upward from the top of a building 96 feet tall with an initial velocity of 80 feet per second. The height s (in feet) of the ball above the ground after t seconds is $s = 96 + 80t - 16t^2$. After how many seconds does the ball strike the ground? That is, after how many seconds is the height equal to zero?

(a) $t = 1$
(b) $t = 2$
(c) $t = 3$
(d) $t = 6$
(e) None of these

1.2 E. The equation $1 - \frac{1}{x} - \frac{12}{x^2} = 0$ has

(a) NO real solutions
(b) exactly ONE real solution, which is POSITIVE
(c) exactly ONE real solution, which is NEGATIVE
(d) exactly TWO real solutions, whose product is -12
(e) exactly TWO real solutions, whose product is $-\frac{1}{12}$

1.4 A. Find the real solutions of the equation $\sqrt{x^2 + 16} = 5$

(a) There are exactly two real roots whose sum is 5
(b) There are exactly two real roots whose product is 5
(c) There are exactly two real roots whose sum is 0
(d) There is exactly one real root, which is positive
(e) There is exactly one real root, which is negative

1.4 #25. Find the real solutions: $3 + \sqrt{3x + 1} = x$

(a) $x = 1$ or $x = -8$
(b) $x = 1$ or $x = 8$
(c) $x = -8$
(d) $x = 8$
(e) None of these

1.4 #28. The solution to the equation $\sqrt{3x + 7} + \sqrt{x + 2} = 1$ is

(a) $x = -1$
(b) $x = -2$
(c) $x = -1$ or $x = -2$
(d) There is no solution
(e) None of these

1.4 #31. The solution to the equation $\sqrt{3 - 2\sqrt{x}} = \sqrt{x}$ is

(a) $x = 9$

(b) $x = 1$ or $x = 9$

(c) $x = 3$ or $x = -3$

(d) There is no solution

(e) None of these

1.5 A. The solution to the inequality $9x - 5 < 6x + 1$ is

(a) $x < 2$

(b) $x > 2$

(c) $x < \frac{2}{5}$

(d) $x > \frac{2}{5}$

(e) None of these

1.5 #75. The solution set of the inequality $1 < 1 - \frac{1}{2}x < 4$ is the interval

(a) $(-6, 0)$

(b) $(0, 6)$

(c) $[0, 6]$

(d) $[-6, 0]$

(e) None of these

1.5 #80. The solution set of the inequality $x(9x - 5) \leq (3x - 1)^2$ is

(a) $[1, \infty)$

(b) $(-\infty, 1]$

(c) $[\frac{1}{5}, \infty)$

(d) $\{0, \frac{5}{9}, \frac{1}{3}\}$

(e) The empty set

1.5 #87. The solution set of the inequality $0 < (2x - 4)^{-1} < \frac{1}{2}$ is the interval

(a) $(3, \infty)$

(b) $(-\infty, 3)$

(c) $(0, 3)$

(d) $(0, \infty)$

(e) None of these

1.5 B. The solution set of the inequality $0 < (x - 4)^{-1} < \frac{1}{2}$ is the interval

(a) $(0, 2)$

(b) $(0, 6)$

(c) $(6, \infty)$

(d) $(-\infty, 6)$

(e) $(2, 3)$

1.6 A. The solution set of the combined inequality $-1 < 3 - 2x \leq 15$ is

(a) $(-6, 2]$
(b) $[-6, 2)$
(c) $(2, 6]$
(d) $[2, 6)$
(e) $[-13/2, \infty)$

1.6 B. Solve this inequality: $|x - 2| < 3$

(a) $-2 < x < 1$
(b) $-2 < x < 5$
(c) $2 < x < 5$
(d) $0 < x < 1$
(e) None of these

1.6 Example 6. Solve this inequality: $|2x - 5| > 3$

(a) $x < 1$ or $x > 4$
(b) $x < -1$ or $x > 4$
(c) $x < 4$ and $x > 1$
(d) $x < 4$ and $x > -1$
(e) None of these

1.6 C. Find the solution set of this inequality: $|5 - 2x| < 9$

(a) The empty set (no solutions)
(b) $\{x \mid x > 7 \text{ or } x < -2\}$
(c) $(-2, 7)$
(d) $(-2, \infty)$
(e) $(-\infty, 7)$

1.6 D. Find the solution set of this inequality: $|3 - 2x| \geq 7$

(a) $\{x \mid x \leq 5 \text{ or } x \geq -2\}$
(b) $\{ x \mid x \geq 5 \text{ or } x \leq -2\}$
(c) $[-2, 5]$
(d) all real numbers
(e) $(-\infty, -2]$

1.6 #43. The solution set of the inequality $| 2x - 3 | \geq 2$ is

(a) $\{x \mid \frac{1}{2} \leq x \leq \frac{5}{2}\}$
(b) $\{x \mid x \leq -\frac{1}{2} \text{ or } x \geq \frac{5}{2}\}$
(c) $\{x \mid x \leq \frac{1}{2} \text{ or } x \geq \frac{5}{2}\}$
(d) $\{x \mid -\frac{1}{2} \leq x \leq \frac{5}{2}\}$
(e) None of these

1.6 #45. Solve this inequality: $|1 - 4x| - 7 < -2$

(a) $-\frac{3}{2} < x < 1$
(b) $-\frac{3}{2} < x < -1$
(c) $1 < x < \frac{3}{2}$
(d) $-1 < x < \frac{3}{2}$
(e) None of these

1.7 #27. A motorboat maintains a constant speed of 15 miles per hour relative to the water in going 10 miles upstream and then returning. If the total time for the trip is 1.5 hours, then the speed of the current must be

(a) 2 miles per hour
(b) 4 miles per hour
(c) 5 miles per hour
(d) 10 miles per hour
(e) None of these

1.7 A. (see #28) Two cars enter the Florida Turnpike at Commercial Boulevard at 8:00 AM, each heading for Wildwood. One car's average speed is 5 miles per hour more than the other's. The slower car arrives at Wildwood at 11:00 AM, 15 minutes after the other car. What is the average speed of the slower car?

(a) 50 mph
(b) 55 mph
(c) 60 mph
(d) 70 mph
(e) None of these

1.7 #33. Trent can deliver his newspapers in 30 minutes. It takes Lois 20 minutes to do the same route. How long would it take them to deliver the newspapers if they work together?

(a) 50 minutes
(b) 25 minutes
(c) 12 minutes
(d) 10 minutes
(e) None of these

1.7 B. If x gallons of cherry juice costing \$3.00 per gallon are to be combined with y gallons of apple juice costing \$1.00 per gallon to make a fruit juice mix costing \$2.50 per gallon, then what is $\frac{x}{y}$?

(a) 3
(b) 10
(c) $\frac{1}{3}$
(d) $\frac{8}{5}$
(e) It cannot be determined

Answer Key

1.1 A. (d)

1.1 B. (b)

1.1 Example 6. (d)

1.1 #51. (d)

1.1 #59. (e)

1.1 C. (b)

1.1 #96. (e)

1.2 A.(a)

1.2 B. (d)

1.2 C. (b)

1.2 D. (c)

1.2 #51. (e)

1.2 #67. (b)

1.2 #87. (b)

1.2 #105. (c)

1.2 #107. (d)

1.2 E. (d)

1.4 A. (c)

1.4 #25. (d)

1.4 #28. (b)

1.4 #31. (e)

1.5 A. (a)

1.5 #75. (a)

1.5 #80. (b)

1.5 #87. (a)

1.5 B. (c)

1.6 A. (b)

1.6 B. (e)

1.6 Example 6. (a)

1.6 C. (c)

1.6 D. (b)

1.6 #43. (c)

1.6 #45. (d)

1.7 #27. (c)

1.7 A. (b)

1.7 #33. (c)

1.7 B. (a)

Solutions

1.1 A. Solve for x: $7 - 2x = 9 + 3x$

Solution: $7 - 2x = 9 + 3x$ $\quad 7 - 9 = 3x + 2x$ $\quad 5x = -2$ $\quad x = -\frac{2}{5}$

1.1 B. Solve the equation: $1 - \frac{1}{2}x = 6 + x.$

Solution: $1 - \frac{1}{2}x = 6 + x.$ $\quad 2 - x = 12 + 2x.$ $\quad -10 = 3x.$ $\quad x = -\frac{10}{3}.$

1.1 C. Going into the final exam, which will count as two tests, Brooke has test scores of 80, 83, 71, 61, and 89. What score does Brooke need on the final in order to have an average score of 80?

Solution: If x is the final exam score, her average will be $(80+83+71+61+89+2x)/7$, since the final counts double. She needs $(80 + 83 + 71 + 61 + 89 + 2x)/7 = 80$, so she must have $80+83+71+61+89+2x = 560$, which reduces to $384+2x = 560$, or $2x = 176$. The answer is $x = 88$.

1.1 #96. A wool suit, discounted by 30% for a clearance sale, has a price tag of \$399. What was the suit's original price?

Solution: Since the suit has been discounted by 30%, the sale price is 70% of the original price. If we let x be the original price, then we get the equation $0.7x = 399$, and dividing both sides by 0.7 gives the answer: \$570.

1.2 A.Solve for x: $x^2 - 3x + 2 = 0$

Solution: This can be solved by factoring. $x^2 - 3x + 2 = 0$ $\quad (x - 2)(x - 1) = 0$
The only way a product of real numbers can be zero is if one of the numbers is zero, so either $x - 2 = 0$ or $x - 1 = 0$ and this gives us the answer: $x = 1$ or $x = 2$.

1.2 B. Solve for x: $x^2 - 2x = 4$

Solution: Put all terms on the left hand side of the equation and try to factor.
It is hard to see a factorization for $x^2 - 2x - 4 = 0$ so the next choice is to use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 1$, $b = -2$, and $c = -4$. This gives us

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot (-4)}}{2} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm \sqrt{4 \cdot 5}}{2} = \frac{2 \pm \sqrt{4}\sqrt{5}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}.$$

1.2 C. Find the value of a so that $x^2 + ax + \frac{1}{9}$ is a perfect square.

Solution: The general theory is that if you square $x - c$, you get $x^2 + 2cx + c^2$. If you know the last coefficient, then to get the middle one you take the square root and multiply by 2. In this problem, take the square root of $\frac{1}{9}$, which gives you $\frac{1}{3}$, and double it. The answer: $a = \frac{2}{3}$. Check: $(x - \frac{1}{3})^2 = (x - \frac{1}{3})(x - \frac{1}{3}) = x^2 + \frac{2}{3}x + \frac{1}{9}$

1.2 D. Find the value of k so that $x^2 - \frac{3}{2}x + k$ is a perfect square.

Solution: This time (see the previous solution) we know the middle coefficient and need to find the last one. Remember the form $x^2 + 2cx + c^2$. We need to divide the middle coefficient by 2, and then square to get the last coefficient. Divide $\frac{3}{2}$ by 2 (multiplying by $\frac{1}{2}$ gives the same answer) to get $\frac{3}{2}$, and then squaring gives us the answer: $\frac{9}{16}$.
Check: $(x - \frac{3}{4})^2 = (x - \frac{3}{4})(x - \frac{3}{4}) = x^2 - \frac{6}{4}x + \frac{9}{16}$

1.2 E. The equation $1 - \dfrac{1}{x} - \dfrac{12}{x^2} = 0$ has

Solution: We cannot have $x = 0$, so we can multiply through by x^2, to get $x^2 - x - 12 = 0$. This equation can be solved by factoring, since $x^2 - x - 12 = (x-4)(x+3)$, so we get $x = 4$ or $x = -3$. Answer: there are exactly TWO real solutions, whose product is -12.

1.4 A. Find the real solutions of the equation $\sqrt{x^2 + 16} = 5$

Solution: $\sqrt{x^2 + 16} = 5 \qquad x^2 + 16 = 25 \qquad x^2 = 9 \qquad x = \pm 3$

Check (since the equation was squared): $\sqrt{(\pm 3)^2 + 16} = \sqrt{9 + 16} = \sqrt{25} = 5$

1.4 #28. The solution to the equation $\sqrt{3x+7} + \sqrt{x+2} = 1$ is

Solution: First move $\sqrt{x+2}$ to the other side of the equation, then square both sides. At the end, you must check the answers carefully because squaring the equation may have introduced an extra solution.

$\sqrt{3x+7} + \sqrt{x+2} = 1 \qquad \sqrt{3x+7} = 1 - \sqrt{x+2} \qquad (\sqrt{3x+7})^2 = (1 - \sqrt{x+2})^2$

$3x + 7 = (1 - \sqrt{x+2})(1 - \sqrt{x+2}) = 1 - 2\sqrt{x+2} + (\sqrt{x+2})^2 = 1 - 2\sqrt{x+2} + x + 2$

$3x + 7 = x + 3 - 2\sqrt{x+2} \qquad 2x + 4 = 2\sqrt{x+2} \qquad x + 2 = \sqrt{x+2}$

Now we need to square both sides again. $(x+2)^2 = (\sqrt{x+2})^2 \qquad x^2 + 4x + 4 = x + 2$

$x^2 + 3x + 2 = 0 \qquad (x+2)(x+1) = 0 \qquad x + 2 = 0$ or $x + 1 = 0 \qquad x = -2$ or $x = -1$

Check these answers: $\sqrt{3(-2)+7} + \sqrt{(-2)+2} = \sqrt{1} + \sqrt{0} = 1$, so -2 is a solution.

$\sqrt{3(-1)+7} + \sqrt{(-1)+2} = \sqrt{4} + \sqrt{1} = 3$, so -1 is *not* a solution. Final answer: $x = -2$.

1.5 A. The solution to the inequality $9x - 5 < 6x + 1$ is $\{x \mid x < 2\}$.

Solution: $9x - 5 < 6x + 1 \quad 9x - 6x < 1 + 5 \quad 3x < 6 \quad x < 2$

1.5 #80. The solution set of the inequality $x(9x-5) \leq (3x-1)^2$ is $(-\infty, 1]$.

Solution: $x(9x-5) \leq (3x-1)^2 \qquad 9x^2 - 5x \leq 9x^2 - 6x + 1 \qquad x \leq 1$ Answer: $(-\infty, 1]$

1.5 B. The solution set of the inequality $0 < (x-4)^{-1} < \frac{1}{2}$ is

Solution: First note that we must have $x - 4 > 0$. Then we can invert the inequality $\dfrac{1}{x-4} < \dfrac{1}{2}$ to get $x - 4 > 2$. (Remember that inverting the terms reverses the inequality. For example, $\frac{1}{4} < \frac{1}{2}$, but $4 > 2$.) The final answer is $(6, \infty)$.

1.6 A. The solution set of the combined inequality $-1 < 3 - 2x \leq 15$ is

Solution: $-1 < 3 - 2x \leq 15$ $\quad -4 < -2x \leq 12$ $\quad 4 > 2x \geq -12$ $\quad 2 > x \geq -6$
Now $-6 \leq x < 2$, so the final answer (in interval form) is $[-6, 2)$.

1.6 B. Solve this inequality: $|x-2| < 3$

Solution: $|x-2| < 3$ $\quad -3 < x - 2 < 3$ $\quad -1 < x < 5$

1.6 C. Find the solution set of this inequality: $|5-2x| < 9$

Solution: $|5-2x| < 9$ $\quad -9 < 5 - 2x < 9$ $\quad -14 < -2x < 4$ $\quad -4 < 2x < 14$
$-2 < x < 7$ Answer (in interval form): $(-2, 7)$

1.6 D. Find the solution set of this inequality: $|3-2x| \geq 7$

Solution: We need to replace $|3-2x| \geq 7$ with two inequalities: $3-2x \geq 7$ or $3-2x \leq -7$.
$-2x \geq 4$ or $-2x \leq -10$ $\quad x \leq -2$ or $x \geq 5$ $\quad$ In set notation: $\{\ x \mid \ x \geq 5 \text{ or } x \leq -2\}$

1.7 A. (see #28) Two cars enter the Florida Turnpike at Commercial Boulevard at 8:00 AM, each heading for Wildwood. One car's average speed is 5 miles per hour more than the other's. The slower car arrives at Wildwood at 11:00 AM, 15 minutes after the other car. What is the average speed of the slower car?

Solution: Let x be the average speed of the slower car. Use the fact that both cars travel the same distance and the formula $rate \cdot time = distance$, with distance in miles and time in hours. Distance for the slower car: $3x$. Distance for the faster car: $(2\frac{3}{4})(x+5)$.
We get this equation: $3x = (2\frac{3}{4})(x+5)$.
Solve: $3x = (2\frac{3}{4})x + (\frac{11}{4})(5)$ $\quad (3 - 2\frac{3}{4})x = \frac{55}{4}$ $\quad \frac{1}{4}x = \frac{55}{4}$ $\quad x = 55$

1.7 B. If x gallons of cherry juice costing \$3.00 per gallon are to be combined with y gallons of apple juice costing \$1.00 per gallon to make a fruit juice mix costing \$2.50 per gallon, then what is $\dfrac{x}{y}$?

Solution: The equation that describes the total cost of the mixture is $3x + y = 2.5(x+y)$.
Divide both sides by y to find the ratio: $\dfrac{3x}{y} + \dfrac{y}{y} = \dfrac{2.5x}{y} + \dfrac{2.5y}{y}$.
Simplify: $(3)\dfrac{x}{y} + 1 = (2.5)\dfrac{x}{y} + 2.5$ $\quad (.5)\dfrac{x}{y} = 1.5$ $\quad \dfrac{x}{y} = 3$