Chapter 2

Graphs

Section summaries

Section 2.1 The Distance and Midpoint Formulas

You need to know the **distance formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the **midpoint formula**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) .$$

The distance formula comes from the Pythagorean theorem (review page 30); you may also need to use the Pythagorean theorem to verify that three points are the vertices of a right triangle.

Review problems: p161 #19,29,35,45

Section 2.2 Graphs of Equations

Review the procedure for finding x and y-intercepts on page 166. Review the tests for symmetry on page 168. A function is **even** precisely when its graph is symmetric with respect to the y-axis; it is **odd** precisely when its graph is symmetric with respect to the origin. (Compare the tests on page 168 to the tests on pages 231 and 232.)

Review problems: p171 #41,43,63,65,67

Section 2.3 Lines

The **slope** of the line segment joining two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \ ,$$

assuming that $x_1 \neq x_2$. The equation of the line through (x_1, y_1) with slope m is

$$y = m(x - x_1) + y_1 \ ,$$

the **point-slope form**. The equation of the line with slope m and y-intercept b is

$$y = mx + b \ ,$$

the **slope-intercept form**. To find the slope of a line in **general form** $Ax + By = C$, put it into the slope-intercept form so you can just read off the slope. Remember that a positive slope means that the graph goes up (from left to right) and a negative slope means that the graph heads down.

Two different lines $y = m_1 x + b_1$ and $y = m_2 x + b_2$ are **parallel** when $m_2 = m_1$, and **perpendicular** when $m_2 = -\dfrac{1}{m_1}$ (or, equivalently, when $m_1 m_2 = -1$).

Review problems: p185 #21,29,57,85,87,113,115,119,131

Section 2.4 Circles

The **standard form** of an equation of a circle with radius r and center (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2 \ .$$

If you are given an equation in the **general form** $x^2 + y^2 + ax + by + c = 0$, you can complete the square to put it into the standard form.

Review problems: p193 #15,19,29,59

Sample Questions

2.1 A. Find the distance between the points $(2, 5)$ and $(4, -3)$.

(a) $2\sqrt{2}$
(b) 10
(c) $\sqrt{10}$
(d) $2\sqrt{17}$
(e) 68

2.1 B. Find the distance between the points $(-1, -3)$ and $(2, 1)$.

(a) 1
(b) $\sqrt{5}$
(c) $\sqrt{17}$
(d) 25
(e) None of these

2.1 C. The midpoint of the line segment joining the points $(1, 6)$ and $(-3, 4)$ is

(a) $(\frac{1}{2}, \frac{7}{2})$
(b) $(\frac{7}{2}, \frac{1}{2})$
(c) $(-1, 5)$
(d) $(-2, -1)$
(e) $(16, 4)$

2.1 #48. Find all points on the y-axis that are 5 units from the point $(4, 4)$.

(a) $(-1, 0)$ and $(-7, 0)$
(b) $(0, 5)$ and $(0, 5)$
(c) $(0, 1)$ and $(0, 7)$
(d) $(0, -1)$ and $(0, -7)$
(e) None of these

2.2 #25. The graph of the line with equation $2x + 3y = 6$ has

(a) x-intercept $(3, 0)$ and y-intercept $(0, 2)$
(b) x-intercept $(2, 0)$ and y-intercept $(0, 3)$
(c) x-intercept $(2, 0)$ and y-intercept $(0, 6)$
(d) x-intercept $(6, 0)$ and y-intercept $(0, 3)$
(e) x-intercept $(6, 0)$ and y-intercept $(0, 2)$

2.2 #59. Find the x-intercepts of the graph of the equation $x^2 + y - 9 = 0$.

(a) The x-intercepts are $\sqrt{3}$ and $-\sqrt{3}$
(b) The only x-intercept is -9
(c) The x-intercepts are 3 and -3
(d) The only x-intercept is 3
(e) None of these

2.2 #61. The graph of the equation $9x^2 + 4y^2 = 36$ has

(a) x-intercept $(0, 0)$ and y-intercept $(0, 0)$

(b) x-intercept $(2, 0)$ and y-intercept $(0, 3)$

(c) x-intercept $(3, 0)$ and y-intercept $(0, 2)$

(d) x-intercepts $(2, 0)$ and $(-2, 0)$ and y-intercepts $(0, 3)$ and $(0, -3)$

(e) x-intercepts $(3, 0)$ and $(-3, 0)$ and y-intercepts $(0, 2)$ and $(0, -2)$

2.2 #69. The graph of $y = \dfrac{-x^3}{x^2 - 9}$ is symmetric with respect to

(a) the x-axis and y-axis, but NOT the origin.

(b) the origin, but NOT the x-axis or y-axis.

(c) the x-axis and the origin, but NOT the y-axis.

(d) the y-axis and origin, but NOT the x-axis.

(e) the x-axis, the y-axis and the origin.

2.3 A. The equation of the vertical line passing through the point $(4, 7)$ is

(a) $x = 4$

(b) $x = 7$

(c) $y = 4$

(d) $y = 7$

(e) $4x = 7y$

2.3 B. Find the slope of the line through the points $(-3, -1)$ and $(1, 7)$.

(a) 3

(b) -3

(c) $\frac{1}{2}$

(d) 2

(e) None of these

2.3 C. Find an equation for the line through $(0, 3)$ and $(-2, 0)$.

(a) $2x - 3y + 6 = 0$

(b) $3x + 2y - 6 = 0$

(c) $3x - 2y + 6 = 0$

(d) $2x + 3y - 6 = 0$

(e) $3x + 2y + 6 = 0$

2.3 Example 8. Find the slope m and y-intercept b of the equation $2x + 4y = 8$.

(a) $m = \frac{1}{2}$ and $b = 2$

(b) $m = -\frac{1}{2}$ and $b = 2$

(c) $m = 2$ and $b = 4$

(d) $m = -2$ and $b = 4$

(e) None of these

2.3 #49. The equation of the line containing the points $(1,3)$ and $(-1,2)$ is

(a) $y = 2x + 1$

(b) $y = -\frac{1}{2}x + \frac{7}{2}$

(c) $y = \frac{1}{2}x + \frac{5}{2}$

(d) $y = -2x + 5$

(e) This is a vertical line, so there is no equation.

2.3 D. Which of the following is an equation of the line passing through the point $(5,-4)$ and parallel to the line with equation $3x - 5y + 2 = 0$?

(a) $y = 3x - 4$

(b) $y = 3x - 19$

(c) $y = \frac{3}{5}x - 7$

(d) $y = \frac{3}{5}x - 4$

(e) $y = -\frac{5}{3}x - 9$

2.3 #65. Find an equation for the line perpendicular to $y = \dfrac{1}{2}x + 4$ containing $(1,-2)$.

(a) $y = 2x + 4$

(b) $y = -2x - 4$

(c) $y = 2x$

(d) $y = -2x$

(e) None of these

2.3 #67. Find an equation for the line perpendicular to $2x + y = 2$ and containing $(-3,0)$.

(a) $y = 2(x + 3)$

(b) $y = -2(x + 3)$

(c) $y = \frac{1}{2}(x + 3)$

(d) $y = -\frac{1}{2}(x + 3)$

(e) None of these

2.3 E. The line which is perpendicular to the line given by $y = 4x - 3$ and which passes through the point $(0,5)$ also passes through which of the following points?

(a) $(4,0)$

(b) $(4,13)$

(c) $(4,4)$

(d) $(4,6)$

(e) $(4,-11)$

2.3 #97. The graph of the line with equation $\frac{1}{2}x + \frac{1}{3}y = 1$ has

(a) x-intercept $(1/2,0)$ and y-intercept $(0,1/3)$

(b) x-intercept $(1/3,0)$ and y-intercept $(0,1/2)$

(c) x-intercept $(3,0)$ and y-intercept $(0,2)$

(d) x-intercept $(2,0)$ and y-intercept $(0,3)$

(e) None of these

2.4 A. The standard form of the equation of the circle with radius 6 and center $(-3,-6)$ is

(a) $(x+3)^2+(y+6)^2=36$

(b) $(x-3)^2+(y-6)^2=36$

(c) $(x+6)^2+(y+3)^2=36$

(d) $(x-6)^2+(y-3)^2=36$

(e) None of these

2.4 #25. The circle $x^2+y^2-2x+4y-4=0$ has

(a) center $(1,-2)$ and radius 9

(b) center $(1,-2)$ and radius 3

(c) center $(-2,4)$ and radius 16

(d) center $(-1,2)$ and radius 3

(e) center $(-1,2)$ and radius 9

2.4 #29. The graph of the equation $x^2+y^2-x+2y+1=0$ is

(a) a circle with center $(1,-2)$ and radius 1.

(b) a circle with center $(-1,2)$ and radius 1.

(c) a circle with center $(\frac{1}{2},-1)$ and radius 1.

(d) a circle with center $(\frac{1}{2},-1)$ and radius $\frac{1}{4}$.

(e) None of these

2.4 B. The graph of the equation $x^2+y^2-6x+2y+7=0$ is

(a) a circle with center $(3,-1)$ and radius 3.

(b) a circle with center $(3,-1)$ and radius $\sqrt{3}$.

(c) a circle with center $(3,1)$ and radius $\sqrt{7}$.

(d) a circle with center $(1,3)$ and radius $\sqrt{3}$.

(e) None of these

Answer Key

2.1 A. (d)

2.1 B. (e)

2.1 C. (c)

2.1 #48. (c)

2.2 #25. (a)

2.2 #59. (c)

2.2 #61. (d)

2.2 #69. (b)

2.3 A. (a)

2.3 B. (d)

2.3. C. (c)

2.3 Example 8. (b)

2.3 #49. (c)

2.3 D. (c)

2.3 #65. (d)

2.3 #67. (c)

2.3 E. (c)

2.3 #97. (d)

2.4 A. (a)

2.4 #25. (b)

2.4 #29. (e)

2.4 B. (b)

Solutions

2.1 A. Find the distance between the points $(2, 5)$ and $(4, -3)$.
Solution: Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
with $x_2 = 4$, $x_1 = 2$, $y_2 = -3$, $y_1 = 5$.
$d = \sqrt{(4-2)^2 + (-3-5)^2} = \sqrt{(2)^2 + (-8)^2} = \sqrt{4 + 64} = \sqrt{68} = \sqrt{2 \cdot 34} = \sqrt{2 \cdot 2 \cdot 17}$
Answer: $d = 2\sqrt{17}$

2.1 B. Find the distance between the points $(-1, -3)$ and $(2, 1)$.
Solution: $d = \sqrt{(2-(-1))^2 + (1-(-3))^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

2.1 C. The midpoint of the line segment joining the points $(1, 6)$ and $(-3, 4)$ is

Solution: Use the midpoint formula $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, which just averages the x-coordinates and the y-coordinates. $\left(\frac{1-3}{2}, \frac{6+4}{2}\right) = \left(\frac{-2}{2}, \frac{10}{2}\right) = (-1, 5)$

2.1 #48. Find all points on the y-axis that are 5 units from the point $(4, 4)$.

Solution: For a point to be on the y-axis its x-coordinate must be zero. Let $(0, y)$ be the point we are looking for, and use the distance formula: $\sqrt{(0-4)^2 + (y-4)^2} = 5$
Solve for y: $16 + (y-4)^2 = 25$ $(y-4)^2 = 9$ $y - 4 = \pm 3$ $y = 1$ or $y = 7$
The two possible points are $(0, 1)$ and $(0, 7)$.

2.3 A. The equation of the vertical line passing through the point $(4, 7)$ is

Solution: The points on the line all have the same x-coordinate, so the equation is $x = 4$.

2.3 B. Find the slope of the line through the points $(-3, -1)$ and $(1, 7)$.

Solution: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-1)}{1-(-3)} = \frac{8}{4} = 2$

2.3 C. Find an equation for the line through $(0, 3)$ and $(-2, 0)$.

Solution: These are the choices: (a) $2x - 3y + 6 = 0$ (b) $3x + 2y - 6 = 0$
(c) $3x - 2y + 6 = 0$ (d) $2x + 3y - 6 = 0$ (e) $3x + 2y + 6 = 0$
You can do this problem by just substituting into the equations. The point $(0, 3)$ lies on lines (b) and (c), since these are the only equations that satisfy $x = 0$, $y = 3$. Of these two, only (c) satisfies $x = -2$ and $y = 0$, so the answer must be equation (c).

You can also solve the problem by using the point-slope form. The slope is $m = \frac{0-3}{-2-0} = \frac{3}{2}$, and the y-intercept is 3 since the line goes through $(0, 3)$. This gives the equation $y = \frac{3}{2}x + 3$. Multiply through by 2 to get $2y = 3x + 6$, or $0 = 3x - 2y + 6$.

2.3 D. Which of the following is an equation of the line passing through the point $(5, -4)$ and parallel to the line with equation $3x - 5y + 2 = 0$?

Solution: To be parallel to the given line, the slope must be the same. Convert the given equation into point-slopt form: $3x + 2 = 5y$ or $y = \frac{3}{5}x + \frac{2}{5}$. The slope is $\frac{3}{5}$.
The choices are (a) $y = 3x - 4$ (b) $y = 3x - 19$ (c) $y = \frac{3}{5}x - 7$
(d) $y = \frac{3}{5}x - 4$ (e) $y = -\frac{5}{3}x - 9$
Only (c) and (d) have the correct slope. The point $(5, -4)$ lies on line (c).

Knowing the slope, you could also use the point-slope form of the equation of a line:
$y = \frac{3}{5}(x - 5) + (-4)$ $y = \frac{3}{5}x - 3 - 4$ $y = \frac{3}{5}x - 7$

2.3 E. The line which is perpendicular to the line given by $y = 4x - 3$ and which passes through the point $(0, 5)$ also passes through which of the following points?

Solution: The slope of the perpendicular line must be $-\frac{1}{4}$, and its y-intercept is 5 since it passes through $(0, 5)$, so its equation is $y = -\frac{1}{4}x + 5$. If $x = 4$, then $y = 4$, so the answer is (c).

2.4 A. The standard form of the equation of the circle with radius 6 and center $(-3, -6)$ is
Solution: Use the standard form of an equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$.
You get $(x+3)^2 + (y+6)^2 = 36$.

2.4 B. The graph of the equation $x^2 + y^2 - 6x + 2y + 7 = 0$ is
Solution: The answer is found by completing the square.
$x^2 - 6x + \;??\; + y^2 + 2y + \;??\; = -7$
$x^2 - 6x + 9 + y^2 + 2y + 1 = -7 + 9 + 1$
$(x-3)^2 + (y+1)^2 = 3$
$(x-3)^2 + (y-(-1))^2 = (\sqrt{3})^2$ This is a circle with center $(3, -1)$ and radius $\sqrt{3}$.