

Chapter 4

Linear and Quadratic Functions

Section summaries

Section 4.1 Linear Functions and Their Properties

A **linear function** is one of the form

$$f(x) = mx + b ,$$

where m gives the slope of its graph, and b gives the y -intercept of its graph. The slope m measures the rate of growth of the function, so a linear function is increasing if $m > 0$ and decreasing if $m < 0$.

Review problems: p284 #17,21,25,37,43,49

Section 4.2 Building Linear Functions from Data

In this section linear functions are constructed from data presented in various ways.

Review problems: p290 #3,5,7,15,19,21

Section 4.3 Quadratic Functions and Their Properties

The **general form of a quadratic function** is

$$f(x) = a(x - h)^2 + k ,$$

where (h, k) is the vertex of the graph (which is a parabola). You can see from the formula that h gives the left/right shift while k gives the up/down shift. The coefficient a represents a vertical stretch or compression. Since the basic member of this family is $f(x) = x^2$, whose graph opens up, the graph of $f(x) = a(x - h)^2 + k$ will open up if a is positive, and down if a is negative. If the graph opens up, its height is minimum at the vertex; if the graph opens down, its height is maximum at the vertex.

If a quadratic is given in the form $f(x) = ax^2 + bx + c$, then the x -coordinate of its vertex is $x = -\frac{b}{2a}$. Since you already know the quadratic formula, you can remember it as part of the formula:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} .$$

This way of looking at the quadratic formula shows that if the graph has x -intercepts, they occur as points on either side of the line $x = -\frac{b}{2a}$, which is the **axis of symmetry** of the graph. The summary on page 301 explains the steps in graphing a quadratic function.

Review problems: p302 #13,15,27,31,37,43,55,61,81,83

4.3 D. Let $f(x) = 4x^2 - 8x + 3$. Find the x and y -intercepts, if any.

- (a) $(-\frac{3}{2}, 0)$ $(-\frac{1}{2}, 0)$ $(0, 3)$ (d) $(15, 0)$ $(0, 3)$
(b) $(\frac{3}{2}, 0)$ $(\frac{1}{2}, 0)$ $(0, 3)$ (e) None of these
(c) $(-1, 0)$ $(0, 3)$

4.3 #55. Find the equation of the quadratic function whose graph has vertex $(-3, 5)$ and y -intercept -4 .

- (a) $f(x) = -(x - 3)^2 + 5$ (d) $f(x) = (x + 3)^2 + 5$
(b) $f(x) = (x - 3)^2 - 13$ (e) None of these
(c) $f(x) = -(x + 3)^2 + 5$

4.3 #61. Find the minimum value of the function $f(x) = 2x^2 + 12x - 3$.

- (a) -57 (d) 51
(b) -29 (e) None of these
(c) -21

4.3 #62. Find the minimum value of the quadratic function $f(x) = 4x^2 - 8x + 3$.

- (a) -5 (d) 15
(b) -1 (e) 35
(c) 3

4.3 #81. Suppose that the manufacturer of a gas clothes dryer has found that when the unit price is p dollars the revenue R (in dollars) is $R(p) = -4p^2 + 4000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

- (a) $\$4000$ (d) $\$3,000,000$
(b) $\$1,000,000$ (e) None of these
(c) $\$500$

4.3 E. A store selling calculators has found that, when the calculators are sold at a price of p dollars per unit, the revenue R (in dollars) as a function of the price p is $R(p) = -750p^2 + 15000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

- (a) $\$10$ (d) $\$75,000$
(b) $\$100$ (e) None of these
(c) $\$60,000$

Answer Key

4.1 #29. (d)

4.1 #37c. (b)

4.3 A. (b)

4.3 #42 (c)

4.3 B. (d)

4.3 C. (a)

4.3 D. (b)

4.3 #55. (c)

4.3 #61. (c)

4.3 #62. (b)

4.3 #81. (b)

4.3 E. (d)

Solutions

4.3 A. If $f(x)$ is a quadratic function whose graph has the vertex (h, k) , which one is the correct form of the function?

Solution: (b) $f(x) = a(x - h)^2 + k$

4.3 B. Find the vertex of the quadratic function $f(x) = 2x^2 - 4x + 9$.

Solution: (d) The text gives this formula: the x -coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is $x = -\frac{b}{2a}$. In this example, $a = 2$ and $b = -4$, so the vertex occurs at $x = -\frac{-4}{2 \cdot 2} = 1$. Then $f(1) = 2 - 4 + 9 = 7$ gives the y -coordinate.

If you forget the formula, you can always complete the square:

$f(x) = 2x^2 - 4x + 9 = 2(x^2 - 2x) + 9 = 2(x^2 - 2x + 1) + 9 - 2 = 2(x - 1)^2 + 7$
so $h = 1$ and $k = 7$ and the vertex is $(1, 7)$.

4.3 C. Find the axis of symmetry of the graph of $f(x) = 4x^2 - 8x + 3$.

Solution: (a) The axis of symmetry passes through the vertex, which has x -coordinate $-\frac{-8}{2 \cdot 4} = 1$. The axis of symmetry is the line $x = 1$.

Again, if you forget the formula, complete the square:

$f(x) = 4x^2 - 8x + 3 = 4(x^2 - 2x) + 3 = 4(x^2 - 2x + 1) + 3 - 4 = 4(x - 1)^2 - 1$
This shows that the vertex is at $(1, -1)$.

4.3 D. Let $f(x) = 4x^2 - 8x + 3$. Find the x and y -intercepts, if any.

Solution: (b) Since $f(0) = 3$, the y -intercept is $(0, 3)$.

To find the x -intercept, solve $4x^2 - 8x + 3 = 0$. This can be factored as $4x^2 - 8x + 3 = (2x - 1)(2x - 3)$, so $2x - 1 = 0$ or $2x - 3 = 0$, giving the x -intercepts $(\frac{3}{2}, 0)$ and $(\frac{1}{2}, 0)$.

4.3 #62. Find the minimum value of the quadratic function $f(x) = 4x^2 - 8x + 3$.

Solution: (b) The minimum value occurs at the vertex, which has x -coordinate $-\frac{-8}{2 \cdot 4} = 1$. Then $f(1) = -1$ is the minimum height.

4.3 E. A store selling calculators has found that, when the calculators are sold at a price of p dollars per unit, the revenue R (in dollars) as a function of the price p is $R(p) = -750p^2 + 15000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

Solution: (d) The graph is a parabola, opening down, so to find the maximum value we need to find the y -coordinate of the vertex. We get $x = -\frac{b}{2a} = -\frac{15000}{2 \cdot (-750)} = -\frac{15000}{-1500} = 10$. The maximum revenue is $R(10) = -750(10)^2 + 15000(10) = -75000 + 150000 = 75000$.