Chapter 4

Linear and Quadratic Functions

Section summaries

Section 4.1 Linear Functions and Their Properties

A **linear function** is one of the form

$$f(x) = mx + b ,$$

where m gives the slope of its graph, and b gives the y-intercept of its graph. The slope m measures the rate of growth of the function, so a linear function is increasing if $m > 0$ and decreasing if $m < 0$.

Review problems: p284 #17,21,25,37,43,49

Section 4.2 Building Linear Functions from Data

In this section linear functions are constructed from data presented in various ways.

Review problems: p290 #3,5,7,15,19,21

Section 4.3 Quadratic Functions and Their Properties

The **general form of a quadratic function** is

$$f(x) = a(x - h)^2 + k ,$$

where (h, k) is the vertex of the graph (which is a parabola). You can see from the formula that h gives the left/right shift while k gives the up/down shift. The coefficient a represents a vertical stretch or compression. Since the basic member of this family is $f(x) = x^2$, whose graph opens up, the graph of $f(x) = a(x - h)^2 + k$ will open up if a is positive, and down if a is negative. If the graph opens up, its height is minimum at the vertex; if the graph opens down, its height is maximum at the vertex.

If a quadratic is given in the form $f(x) = ax^2 + bx + c$, then the x-coordinate of its vertex is $x = -\frac{b}{2a}$. Since you already know the quadratic formula, you can remember it as part of the formula:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$$

This way of looking at the quadratic formula shows that if the graph has x-intercepts, they occur as points on either side of the line $x = -\frac{b}{2a}$, which is the **axis of symmetry** of the graph. The summary on page 301 explains the steps in graphing a quadratic function.

Review problems: p302 #13,15,27,31,37,43,55,61,81,83

Sample Questions

4.1 #29. Solve $f(x) \leq g(x)$ for $f(x) = 4x - 1$ and $g(x) = -2x + 5$.

(a) $(-\infty, 2)$
(b) $(-\infty, 2]$
(c) $(-\infty, 1)$
(d) $(-\infty, 1]$
(e) None of these

4.1 #37c. The cost (in dollars) of renting a truck is $C(x) = 0.25x+35$, where x is the number of miles driven. If you want the cost to be no more than \$100, what is the maximum number of miles you can drive?

(a) 60
(b) 260
(c) 400
(d) 540
(e) None of these

4.3 A. If $f(x)$ is a quadratic function whose graph has the vertex (h, k), which one is the correct form of the function?

(a) $f(x) = a(x - h) + k$
(b) $f(x) = a(x - h)^2 + k$
(c) $f(x) = a(x - k)^2 + h$
(d) $f(x) = \sqrt{r^2 - (x - h)^2} + k$
(e) $f(x) = a(x - k)^3 + h$

4.3 #42 If $f(x) = x^2 - 2x - 3$, then the vertex of the graph of $f(x)$ is

(a) (-2,5)
(b) (-1,0)
(c) (1,-4)
(d) $(2, -3)$
(e) None of these

4.3 B. Find the vertex of the quadratic function $f(x) = 2x^2 - 4x + 9$.

(a) $(0, 9)$
(b) $(1, 1)$
(c) $(-1, 7)$
(d) $(1, 7)$
(e) None of these

4.3 C. Find the axis of symmetry of the graph of $f(x) = 4x^2 - 8x + 3$.

(a) $x = 1$
(b) $x = -1$
(c) $x = 2$
(d) $x = -2$
(e) None of these

4.3 D. Let $f(x) = 4x^2 - 8x + 3$. Find the x and y-intercepts, if any.

(a) $(-\frac{3}{2}, 0)$ $(-\frac{1}{2}, 0)$ $(0, 3)$

(b) $(\frac{3}{2}, 0)$ $(\frac{1}{2}, 0)$ $(0, 3)$

(c) $(-1, 0)$ $(0, 3)$

(d) $(15, 0)$ $(0, 3)$

(e) None of these

4.3 #55. Find the equation of the quadratic function whose graph has vertex $(-3, 5)$ and y-intercept -4.

(a) $f(x) = -(x-3)^2 + 5$

(b) $f(x) = (x-3)^2 - 13$

(c) $f(x) = -(x+3)^2 + 5$

(d) $f(x) = (x+3)^2 + 5$

(e) None of these

4.3 #61. Find the minimum value of the function $f(x) = 2x^2 + 12x - 3$.

(a) -57

(b) -29

(c) -21

(d) 51

(e) None of these

4.3 #62. Find the minimum value of the quadratic function $f(x) = 4x^2 - 8x + 3$.

(a) -5

(b) -1

(c) 3

(d) 15

(e) 35

4.3 #81. Suppose that the manufacturer of a gas clothes dryer has found that when the unit price is p dollars the revenue R (in dollars) is $R(p) = -4p^2 + 4000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

(a) \$4000

(b) \$1,000,000

(c) \$500

(d) \$3,000,000

(e) None of these

4.3 E. A store selling calculators has found that, when the calculators are sold at a price of p dollars per unit, the revenue R (in dollars) as a function of the price p is $R(p) = -750p^2 + 15000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

(a) \$10

(b) \$100

(c) \$60,000

(d) \$75,000

(e) None of these

Answer Key

4.1 #29. (d)

4.1 #37c. (b)

4.3 A. (b)

4.3 #42 (c)

4.3 B. (d)

4.3 C. (a)

4.3 D. (b)

4.3 #55. (c)

4.3 #61. (c)

4.3 #62. (b)

4.3 #81. (b)

4.3 E. (d)

Solutions

4.3 A. If $f(x)$ is a quadratic function whose graph has the vertex (h, k), which one is the correct form of the function?

Solution: (b) $f(x) = a(x-h)^2 + k$

4.3 B. Find the vertex of the quadratic function $f(x) = 2x^2 - 4x + 9$.

Solution: (d) The text gives this formula: the x-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is $x = -\frac{b}{2a}$. In this example, $a = 2$ and $b = -4$, so the vertex occurs at $x = -\frac{-4}{2 \cdot 2} = 1$. Then $f(1) = 2 - 4 + 9 = 7$ gives the y-coordinate.

If you forget the formula, you can always complete the square:
$f(x) = 2x^2 - 4x + 9 = 2(x^2 - 2x) + 9 = 2(x^2 - 2x + 1) + 9 - 2 = 2(x-1)^2 + 7$
so $h = 1$ and $k = 7$ and the vertex is $(1, 7)$.

4.3 C. Find the axis of symmetry of the graph of $f(x) = 4x^2 - 8x + 3$.

Solution: (a) The axis of symmetry passes through the vertex, which has x-coordinate $-\frac{-8}{2 \cdot 4} = 1$. The axis of symmetry is the line $x = 1$.

Again, if you forget the formula, complete the square:
$f(x) = 4x^2 - 8x + 3 = 4(x^2 - 2x) + 3 = 4(x^2 - 2x + 1) + 3 - 4 = 4(x-1)^2 - 1$
This shows that the vertex is at $(1, -1)$.

4.3 D. Let $f(x) = 4x^2 - 8x + 3$. Find the x and y-intercepts, if any.

Solution: (b) Since $f(0) = 3$, the y-intercept is $(0, 3)$.

To find the x-intercept, solve $4x^2 - 8x + 3 = 0$. This can be factored as $4x^2 - 8x + 3 = (2x-1)(2x-3)$, so $2x - 1 = 0$ or $2x - 3 = 0$, giving the x-intercepts $(\frac{3}{2}, 0)$ and $(\frac{1}{2}, 0)$.

4.3 #62. Find the minimum value of the quadratic function $f(x) = 4x^2 - 8x + 3$.

Solution: (b) The minimum value occurs at the vertex, which has x-coordinate $-\frac{-8}{2\cdot 4} = 1$. Then $f(1) = -1$ is the minimum height.

4.3 E. A store selling calculators has found that, when the calculators are sold at a price of p dollars per unit, the revenue R (in dollars) as a function of the price p is $R(p) = -750p^2 + 15000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

Solution: (d) The graph is a parabola, opening down, so to find the maximum value we need to find the y-coordinate of the vertex. We get $x = -\frac{b}{2a} = -\frac{15000}{2\cdot(-750)} = -\frac{15000}{-1500} = 10$. The maximum revenue is $R(10) = -750(10)^2 + 15000(10) = -75000 + 150000 = 75000$.