Chapter 6

Exponential and Logarithmic Functions

Section summaries

Section 6.1 Composite Functions

Some functions are constructed in several steps, where each of the individual steps is a function. For example, you would evaluate $h(x) = (2x+3)^4$ by first computing $g(x) = 2x+3$ and then raising it to the 4th power. This is expressed mathematically by writing $h(x) = f(g(x))$, where $g(x) = 2x+3$ and $f(x) = x^4$. Here one formula is substituted into another, giving a **composite function**.

See page 402 for the definition of a composite function; see page 406 for an important application to calculus.

To find the domain of a composite function $f(g(x))$, start with the domain of $g(x)$. (The domain of $f(g(x))$ is always contained in the domain of g.) Then, depending on the formula you get for $f(g(x))$, you might need to exclude some more values.

Review problems: p407 #13,21,35,51,63,65

Section 6.2 Inverse Functions

The inverse of a function is like a "reverse look-up" function. Usually we use a formula $y = f(x)$ to find y when x is given. What about the reverse? Given y, how can you find x? This is the job of the inverse function. To give a definition of an inverse function, we use the notion of a composite function.

The function $g(x)$ is the **inverse** of $f(x)$ if $f(g(x)) = x$ and $g(f(x)) = x$, for all x. These equations say that g does exactly reverse of f. A good example to think of is $f(x) = \sqrt[3]{x}$ and $g(x) = x^3$.

When g is the inverse of f, we usually write $g = f^{-1}$, and read this as "g equals f inverse". See the box at the top of page 414 for the basic relationship used to define of f^{-1}:

$$f^{-1}(f(x)) = x \quad \text{and} \quad f(f^{-1}(x)) = x\ .$$

Not all functions have an inverse function. If two x-values produce the same y-value in the formula $y = f(x)$, then given y there is no unique way to recover x. In order to have an inverse, the graph of $y = f(x)$ must pass the horizontal line test (see page 411). If $y = f(x)$ passes the test, then we simply interchange x and y in the formula $y = f(x)$, and solve for y to get the formula for the inverse. See page 416 for this procedure that actually finds the formula for the inverse of a function.

A function and its inverse are closely connected. The domain of f^{-1} is the range of f; the range of f^{-1} is the domain of f (see page 413). The graph of f^{-1} is the mirror image of the graph of f, in the line $y = x$. This happens since if the point (x, y) is on the graph of f, then the symmetric point (y, x) must be on the graph of f^{-1} (see page 415).

Review problems: p420 #35,39,43,49,57,81

Section 6.3 Exponential Functions

An **exponential function** is one of the form

$$f(x) = a^x$$

where a is a positive real number and $a \neq 1$. (We will usually assume that $a > 1$.) The domain of an exponential function is the set of all real numbers. Its graph has the x-axis as a horizontal asymptote. The points $(0, 1)$, $(1, a)$, and $(-1, 1/a)$ are easy ones to plot. Note that if x increases by 1, then $f(x)$ is multiplied by a, since

$$f(x + 1) = a^{x+1} = a^x \cdot a = af(x)$$

(see the theorem on page 425).

You need to be familiar with the basic shape of an exponential function. See Figure 18 on page 426 for the graph of $y = 2^x$ and Figure 27 on page 430 for the graph of $y = e^x$. The base e is important because it makes calculations easier when doing calculus. (It is the one exponential function whose graph crosses the y-axis at a 45 degree angle, making the slope of the graph equal to 1 when $x = 0$.) For our class, the only thing you need to remember about e is its approximate value of 2.7 and the fact that the graph of $y = e^x$ lies between the graphs of $y = 2^x$ and $y = 3^x$.

Building on the basic shape of $y = a^x$, we can graph other functions in the family by using transformations (as we did in Section 3.5).

Review problems: p433 #21,25,43,53,65,71,89

Section 6.4 Log Functions

A **logarithmic function** (or **log function** for short) is one of the form

$$f(x) = \log_a(x)$$

where $a > 0$ and $a \neq 1$. If $a = 10$, we usually write $\log(x)$ instead of $\log_{10}(x)$. If $a = e$, we write $\ln(x)$ instead of $\log_e(x)$, and call this the **natural log** function.

The log function $\log_a(x)$ is defined to be the **inverse of the exponential function** a^x. First, this tells us the basic shape of the graph (see Figure 30 on page 440). It also guarantees that the graph has the y-axis as a vertical asymptote, and that the domain of $\log_a(x)$ is $(0, +\infty)$, the same as the range of a^x. Now, when finding the domain of a function, you not only need to watch out for division by zero, or the square root of a negative number, but also for the log of a negative number. All of these are undefined for real numbers.

To express the inverse relationship, we can say that $y = \log_a(x)$ if and only if $x = a^y$ (see the top of page 438). We also have the following equations, which summarize the inverse relationship (see the theorem at the top of page 451):

$$a^{\log_a(x)} = x \quad \text{and} \quad \log_a(a^x) = x \; .$$

These identities are important in solving equations that involve logs. For example, to solve the equation $\log_2(2x + 1) = 3$ we need to simplify the left hand side. Since $2^{\log_2(2x+1)} = 2x + 1$, the first step is to make both sides of the equation into an exponent with base 2, to get $2^{\log_2(2x+1)} = 2^3$, which simplifies to $2x + 1 = 8$. To solve the equation $\ln(e^{-2x}) = 8$, just note that the left hand side is equal to $-2x$, so the equation simplifies immediately to $-2x = 8$. To solve the equation $e^{2x+5} = 8$, we need to get rid of the base e on the left hand side. This is done by substituting both sides into the natural log function, to get $\ln(e^{2x+5}) = \ln(8)$, or simply $2x + 5 = \ln 8$.

Review problems: p446 #21,33,37,43,63,71,81,89,99

Section 6.5 Properties of Logarithms

Since logs represent exponents, they should behave like exponents. For example, if we write two numbers M and N in scientific notation as powers of 10, then to multiply M and N we only need to add the exponents. To find the square root of M, we only need to divide the exponent of M by 2. The crucial properties of logs are summarized in the following equations (see page 451 and 452).

$$\log_a(MN) = \log_a(M) + \log_a(N) \qquad \log_a(M/N) = \log_a(M) - \log_a(N) \qquad \log_a(M^r) = r\log_a(M)$$

There is also a formula to change the base: $\log_a(M) = \dfrac{\log_b(M)}{\log_b(a)}$. (See the section summary on page 456.)

Review problems: p457 #13,17,29,41,49,57,83,85

Section 6.6 Log and exponential equations

In this section, the properties of logarithms are used to solve various kinds of equations.

Review problems: p463 #17,31,35,77,81

Sample Questions

6.1 #11a. Let $f(x) = 2x$ and $g(x) = 3x^2 + 1$. Find $(f \circ g)(4)$.

(a) 337
(b) 193
(c) 98
(d) $24x^3 + 8x$
(e) None of these

6.1 #11. Let $f(x) = 2x$ and $g(x) = 3x^2 + 1$. Find the composite function $(g \circ f)(x)$.

(a) $12x^2 + 1$
(b) $6x^2 + 2$
(c) $6x^2 + 1$
(d) $6x^3 + 2x$
(e) $6x^3 + 1$

6.1 #15a. Let $f(x) = \sqrt{x}$ and let $g(x) = 2x$. Find $(f \circ g)(4)$.

(a) $\sqrt{2}$
(b) $2\sqrt{2}$
(c) 4
(d) 16
(e) None of these

6.1 A. Let $f(x) = 2x^2 + 1$ and let $g(x) = x + 3$. Find the composite function $(f \circ g)(x)$.

(a) $2x^2 + 18$
(b) $2x^2 + 19$
(c) $2x^2 + 12x + 18$
(d) $2x^2 + 12x + 19$
(e) None of these

6.1 #31b. Let $f(x) = 3x + 1$ and $g(x) = x^2$. Find the composite function $(g \circ f)(x)$.

(a) $x^2 + 3x + 1$
(b) $9x^2 + 1$
(c) $3x^3 + x^2$
(d) $9x^2 + 6x + 1$
(e) None of these

6.1 Example 4a. Find the domain of $f \circ g$ if $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{4}{x-1}$.

(a) $\{x \mid x \neq \pm 1\}$
(b) $\{x \mid x \neq 1\}$
(c) $\{x \mid x \neq -1\}$
(d) $\{x \mid x \neq -2\}$
(e) None of these

6.1 #35a1. Let $f(x) = \dfrac{3}{x-1}$ and $g(x) = \dfrac{2}{x}$. Find the composite function $(f \circ g)(x)$.

(a) $\dfrac{6}{x^2 - x}$

(b) $\dfrac{2x-2}{3}$

(c) $3x$

(d) $\dfrac{3x}{2-x}$

(e) None of these

6.1 #35a2. Let $f(x) = \dfrac{3}{x-1}$ and $g(x) = \dfrac{2}{x}$. Find the domain of $f \circ g$.

(a) $\{x \mid x \neq 1, x \neq 0\}$

(b) $\{x \mid x \neq 1, x \neq 0, x \neq 2\}$

(c) $\{x \mid x \neq 2\}$

(d) $\{x \mid x \neq 2, x \neq 1\}$

(e) $\{x \mid x \neq 2, x \neq 0\}$

6.1 #36. Find the domain of $f \circ g$ if $f(x) = \dfrac{1}{x+3}$ and $g(x) = \dfrac{-2}{x}$.

(a) $\{x \mid x \neq -3\}$

(b) $\{x \mid x \neq 2/3\}$

(c) $\{x \mid x \neq 0 \text{ and } x \neq -2/3\}$

(d) $\{x \mid x \neq 0 \text{ and } x \neq 2/3\}$

(e) None of these

6.1 #61. If $f(x) = 2x^2 + 5$ and $g(x) = 3x + a$, find a so that the y-intercept of $f \circ g$ is 23.

(a) $a = 8$

(b) $a = -8$

(c) $a = \pm 3\sqrt{2}$

(d) $a = \pm 3$

(e) None of these

6.1 #63a. Find $f \circ g$ for $f(x) = ax + b$ and $g(x) = cx + d$.

(a) $(f \circ g)(x) = acx^2 + (b+c)x + bd$

(b) $(f \circ g)(x) = acx + bc + d$

(c) $(f \circ g)(x) = acx + ad + b$

(d) $(f \circ g)(x) = acx + b + d$

(e) None of these

6.1 B. If $f(x) = 3x^2 - 7$ and $g(x) = 2x + a$, find a so that the graph of $f \circ g$ crosses the y-axis at 5.

(a) $a = \pm 2$

(b) $a = \pm 2\sqrt{3}$

(c) $a = \pm 3$

(d) $a = \pm 5$

(e) None of these

6.2 A. If $f(x)$ has an inverse, and $(2, -\frac{1}{2})$ is on the graph of $f(x)$, then what point is on the graph of $f^{-1}(x)$?

(a) $(\frac{1}{2}, -2)$
(b) $(-2, -\frac{1}{2})$
(c) $(-2, \frac{1}{2})$
(d) $(-\frac{1}{2}, 2)$
(e) $(2, -\frac{1}{2})$

6.2 #49. The inverse of the function $f(x) = 4x + 2$ is

(a) $f^{-1}(x) = \dfrac{x+2}{4}$
(b) $f^{-1}(x) = \dfrac{x+4}{2}$
(c) $f^{-1}(x) = \frac{1}{4}x - \frac{1}{2}$
(d) $f^{-1}(x) = \frac{1}{2}x - \frac{1}{4}$
(e) None of these

6.2 B. The function $f(x) = \sqrt{x-2}$, for $x \geq 2$, is a one-to-one function. Find the inverse function f^{-1}.

(a) $f^{-1}(x) = x^2 + 2$, for $x \geq 0$
(b) $f^{-1}(x) = x^2 + 2$, for $x \geq 2$
(c) $f^{-1}(x) = x^2 + 2$, for all x
(d) $f^{-1}(x) = -\sqrt{x-2}$, for $x \geq 2$
(e) $f^{-1}(x) = \dfrac{1}{\sqrt{x-2}}$, for $x > 2$

6.2 #57a. The function $f(x) = \dfrac{1}{x-2}$ is a one-to-one function. Find the inverse function.

(a) $f^{-1}(x) = \dfrac{1}{x} - \frac{1}{2}$
(b) $f^{-1}(x) = \dfrac{1}{x} + 2$
(c) $f^{-1}(x) = \dfrac{1}{x-2}$
(d) $f^{-1}(x) = x - 2$
(e) None of these

6.2 #57b. Find the range of the function $f(x) = \dfrac{1}{x-2}$. (See the previous problem.)

(a) $\{y \mid y \neq 2\}$
(b) $\{y \mid y \neq -1/2\}$
(c) $\{y \mid y \neq 0\}$
(d) All real numbers
(e) None of these

6.2 #59a. The function $f(x) = \dfrac{2}{x+3}$ is a one-to-one function. Find the inverse function.

(a) $f^{-1}(x) = \dfrac{2}{x} - \frac{2}{3}$

(b) $f^{-1}(x) = \dfrac{2}{x} + \frac{2}{3}$

(c) $f^{-1}(x) = \dfrac{2}{x} - 3$

(d) $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

(e) None of these

6.2 #59b. Find the range of the function $f(x) = \dfrac{2}{x+3}$. (See the previous problem.)

(a) $\{y \mid y \neq 0\}$

(b) $\{y \mid y \neq 2/3\}$

(c) $\{y \mid y \neq 1/2\}$

(d) All real numbers

(e) None of these

6.2 #60. The function $f(x) = \dfrac{4}{2-x}$, for $x \neq 2$, is a one-to-one function. Find the inverse function f^{-1}.

(a) $f^{-1}(x) = 2 - \dfrac{4}{x}$

(b) $f^{-1}(x) = \dfrac{1}{2} - \dfrac{1}{4}x$

(c) $f^{-1}(x) = \dfrac{-4}{2-x}$

(d) $f^{-1}(x) = \dfrac{4}{x} - 2$

(e) None of these

6.2 #63a. The function $f(x) = \dfrac{2x}{3x-1}$ is a one-to-one function. Find the inverse f^{-1}.

(a) $f^{-1}(x) = \dfrac{3x-1}{2x}$

(b) $f^{-1}(x) = \dfrac{x}{3}$

(c) $f^{-1}(x) = \dfrac{x}{3x-2}$

(d) $f^{-1}(x) = \dfrac{x}{2-3x}$

(e) None of these

6.2 #63b. Find the range of the function $f(x) = \dfrac{2x}{3x-1}$. (See the previous problem.)

(a) all real numbers except $\frac{1}{3}$

(b) all real numbers except $-\frac{1}{3}$

(c) all real numbers except $\frac{2}{3}$

(d) all real numbers except $-\frac{2}{3}$

(e) all real numbers except 0

6.2 C. The function $f(x) = 3x - 2$ is a one-to-one function. Find the inverse function f^{-1}.

(a) $f^{-1}(x) = \dfrac{1}{3x-2}$

(b) $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

(c) $f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

(d) $f^{-1}(x) = \frac{1}{3}x + \frac{2}{3}$

(e) None of these

6.2 #94. The period T of a simple pendulum is $T = 2\pi\sqrt{\dfrac{x}{g}}$, where x is its length and g is a constant (the acceleration due to gravity). Solve for x as a function of T.

(a) $x = 2\pi\sqrt{\dfrac{T}{g}}$

(b) $x = \dfrac{gT}{2\pi}$

(c) $x = \dfrac{gT^2}{2\pi}$

(d) $x = \dfrac{gT^2}{4\pi^2}$

(e) None of these

6.3 A. Which answer describes the graph of the exponential function $f(x) = e^x$?

(a) The graph goes through $(0, e)$ and decreases as x increases.

(b) The graph goes through $(0, e)$ and increases as x increases.

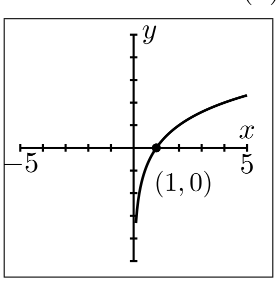
(c) The graph goes through $(0, 1)$ and decreases as x increases.

(d) The graph goes through $(0, 1)$ and increases as x increases.

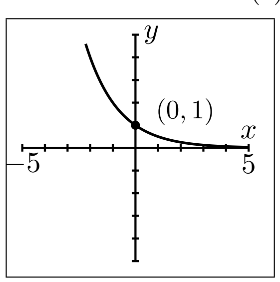
(e) The graph is a straight line through $(1, e)$.

6.3 B. Which of the following is the graph of $y = 2^x$?

(a)



(b)



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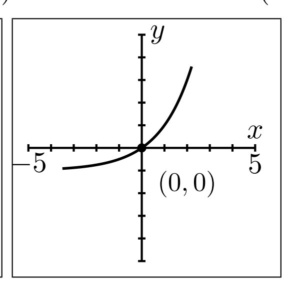
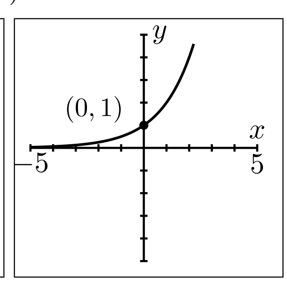
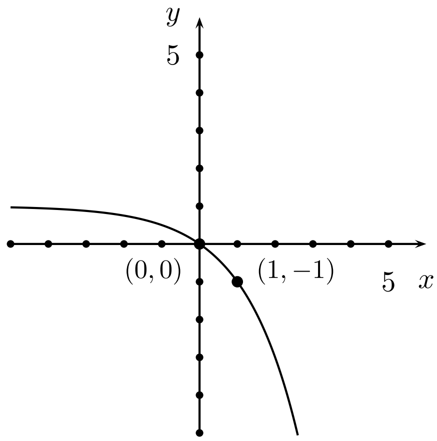


(d)



6.3 C. Which exponential function is represented by this graph?



(a) $f(x) = -2x^2 + x$

(b) $f(x) = 1 - 2^{-x}$

(c) $f(x) = 1 - 2^x$

(d) $f(x) = 1 + 2^x$

(e) $f(x) = 1 + e^x$

6.3 #43. Find the horizontal asymptote of $f(x) = 3^{-x} - 2$.

(a) $y = 2$

(b) $y = 0$

(c) $y = -2$

(d) $x = 0$

(e) $x = -2$

6.3 #52a. The horizontal asymptote of the graph of $y = e^x - 1$ is

(a) $y = -1$

(b) $y = 0$

(c) $y = 1$

(d) $y = e$

(e) There is no horizontal asymptote

6.3 #52b. The vertical asymptote of the graph of $y = e^x - 1$ is

(a) $x = -1$

(b) $x = 0$

(c) $x = 1$

(d) $x = e$

(e) There is no vertical asymptote

6.3 #55. Find the horizontal asymptote of the graph of $f(x) = 2 - e^{-x/2}$.

(a) $y = -2$

(b) $y = 0$

(c) $x = -2$

(d) $x = 0$

(e) None of these

6.3 D. Solve for x: $9^{2x} = 27$

(a) $x = \log_9 27$
(b) $x = \log_3 27$
(c) $x = \frac{4}{3}$
(d) $x = \frac{3}{4}$
(e) None of these

6.3 #61. Solve for x: $\left(\frac{1}{5}\right)^x = \frac{1}{25}$

(a) $x = -2$
(b) $x = -1/2$
(c) $x = 1/2$
(d) $x = 2$
(e) None of these

6.3 #63. Solve for x: $2^{2x-1} = 4$

(a) $x = 0$
(b) $x = -\frac{1}{2}$
(c) $x = 1$
(d) $x = \frac{3}{2}$
(e) There is no solution

6.3 E. Solve for x: $\sqrt{3}^{\,x+2} = \frac{1}{9}$

(a) $x = -4$
(b) $x = -5$
(c) $x = -6$
(d) $x = -3/2$
(e) None of these

6.3 #85. If $3^{-x} = 2$, what does 3^{2x} equal?

(a) 4
(b) -4
(c) $\frac{1}{4}$
(d) $-\frac{1}{4}$
(e) None of these

6.3 #86. If $5^{-x} = 3$, what does 5^{3x} equal?

(a) -9
(b) -1
(c) $-1/9$
(d) $1/9$
(e) None of these

6.4 A. Which answer describes the graph of the logarithmic function $f(x) = \ln x$?

(a) The graph goes through $(0, 1)$ and has $x = 0$ as a vertical asymptote.

(b) The graph goes through $(1, 0)$ and has $x = 0$ as a vertical asymptote.

(c) The graph goes through $(0, 1)$ and has $y = 0$ as a horizontal asymptote.

(d) The graph goes through $(1, 0)$ and has $y = 0$ as a horizontal asymptote.

(e) The graph is a straight line through $(0, 1)$ and $(e, 1)$.

6.4 B. List the properties of the graph of $y = \ln x$.

A: The graph has a vertical asymptote at $x = 1$.
B: The graph has a vertical asymptote at $x = 0$.
C: The graph goes through $(e, 0)$.
D: The graph goes through $(1, 0)$.
E: The graph has a horizontal asymptote.
F: The graph increases as x increases.

(a) A, C, and E

(b) A, D, and E

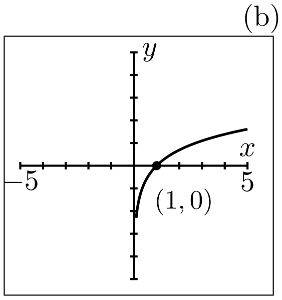
(c) A, C, and F

(d) B, D, and E

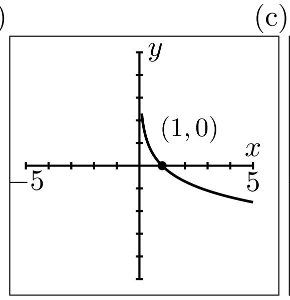
(e) B, D, and F

6.4 C. Which of the following is the graph of $y = \ln x$?

(a)



(b)



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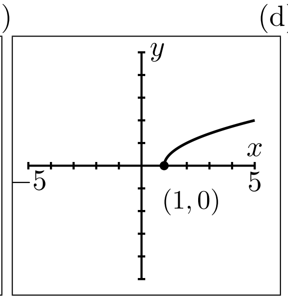

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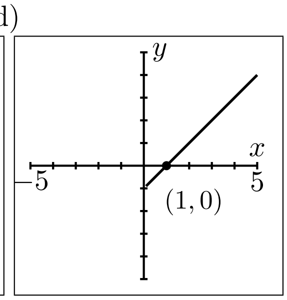


6.4 D. Which of the following pairs of functions are inverses of each other?

(a) $\ln(x)$ and 10^x

(b) $\log(x)$ and e^x

(c) $\ln(x)$ and e^x

(d) $\log_{2.7}(x)$ and e^x

(e) $\log_2(x)$ and $\left(\frac{1}{2}\right)^x$

6.4 E. If $f(x) = \log_3(x)$, what is $f^{-1}(x)$?

(a) $f^{-1}(x) = e^x$

(b) $f^{-1}(x) = 3^x$

(c) $f^{-1}(x) = -\log_3(x)$

(d) $f^{-1}(x) = \dfrac{1}{\log_3(x)}$

(e) $f^{-1}(x) = \log_{\frac{1}{3}}(x)$

6.4 #29. $\log_{\frac{1}{2}} 16 =$

(a) 8
(b) 4
(c) $\frac{1}{4}$
(d) $-\frac{1}{4}$
(e) -4

6.4 #33. $\log_{\sqrt{2}} 4 =$

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

6.4 #35. $\ln \sqrt{e} =$

(a) -1
(b) .5
(c) 1.359
(d) 2.718
(e) None of these

6.4 F. The domain of $f(x) = \log(1 - 5x)$ is

(a) $(\frac{1}{5}, \infty)$
(b) $[\frac{1}{5}, \infty)$
(c) $(-\infty, \frac{1}{5}]$
(d) $(-\infty, \frac{1}{5})$
(e) None of these

6.4 #43. The domain of $f(x) = \ln\left(\dfrac{1}{x+1}\right)$ is

(a) $\{x \mid x \geq -1\}$
(b) $\{x \mid x \neq -1\}$
(c) $\{x \mid x < -1\}$
(d) $\{x \mid x > -1\}$
(e) None of these

6.4 #82. Find the vertical asymptote of the graph of $f(x) = 2 - \log_3(x + 1)$.

(a) $x = -1$
(b) $x = 0$
(c) $x = 1$
(d) $y = 0$
(e) None of these

6.4 G. The equation $\log_\pi x = \frac{1}{2}$ can be written in exponential form as

(a) $x = \left(\frac{1}{2}\right)^\pi$
(b) $x = \pi^{1/2}$
(c) $x^\pi = \frac{1}{2}$
(d) $\pi = x^{1/2}$
(e) $\pi = \left(\frac{1}{2}\right)^x$

6.4 #89. Solve: $\log_2(2x+1) = 3$

(a) $x = 1$
(b) $x = 0$
(c) $x = 3$
(d) $x = 4$
(e) None of these

6.5 #14. $\log_6 4 + \log_6 9 =$

(a) 2
(b) 13/6
(c) $\log_6 13$
(d) $\log_6(4/9)$
(e) None of these

6.5 A. $(\log_2 6)(\log_6 8) =$

(a) 2
(b) 3
(c) $\log_6 4$
(d) $\log_2(4/3)$
(e) None of these

6.5 B. $(\log_3 6)(\log_6 9) =$

(a) $\log_6 3$
(b) $\log_3(3/2)$
(c) 2
(d) 3
(e) None of these

6.5 #29. If $\ln 2 = a$ and $\ln 3 = b$, then $\ln \sqrt[5]{6} =$

(a) $\frac{1}{5}ab$
(b) $\frac{1}{5}(a+b)$
(c) $5ab$
(d) $5(a+b)$
(e) None of these

6.5 #46. $\log\left(\dfrac{x^3\sqrt{x+1}}{(x-2)^2}\right) =$

(a) $3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)$
(b) $3\log x + \frac{1}{2}\log(x+1) + 2\log(x-2)$
(c) $3\log x + \log(x+1) - \log(x-2)$
(d) $3\log x + \log(x+1) + \log(x-2)$
(e) None of these

6.5 #49. $\ln\left(\frac{5x\sqrt{1+3x}}{(x-4)^3}\right) =$

(a) $5\ln x + \ln(1+3x) + \ln(x-4)$

(b) $5\ln x + \ln(1+3x) - \ln(x-4)$

(c) $\ln 5 + \ln x + \frac{1}{2}\ln(1+3x) - 3\ln(x-4)$

(d) $\ln 5 + \ln x + \frac{1}{2}\ln(1+3x) + 3\ln(x-4)$

(e) None of these

6.5 #72. $\log_\pi \sqrt{2} =$

(a) $\frac{1}{2\ln \pi}$

(b) $\frac{\ln 2}{\ln \pi}$

(c) $\frac{\ln 2}{2\ln \pi}$

(d) $\frac{\ln 2}{\ln \pi}$

(e) None of these

6.5 #83. Express y as a function of x: $\ln y = \ln x + \ln(x+1) + \ln C$

(a) $y = 2x + 1 + C$

(b) $y = Cx(x+1)$

(c) $y = Ce^{x(x+1)}$

(d) $y = e^{Cx(x+1)}$

(e) None of these

6.5 #85. Express y as a function of x (the constant C is positive). $\ln y = 3x + \ln C$

(a) $y = \ln(3x) + C$

(b) $y = Ce^{3x}$

(c) $y = C^{3x}$

(d) $y = e^{3x} + C$

(e) None of these

6.5 #87. Solve for y (the constant C is positive): $\ln(y-3) = -4x + \ln C$

(a) $y = 3 - \frac{4}{\ln x} + C$

(b) $y = 3 + C^{-4x}$

(c) $y = 3 + Ce^{-4x}$

(d) $y = 3 + e^{-4x} + C$

(e) None of these

6.6 A. Solve for x: $\ln(x+1) + \ln(x) = \ln(6)$

(a) $x = 1/5$

(b) $x = -3$

(c) $x = -3$ or $x = 2$

(d) $x = 2$

(e) None of these

6.6 B. Solve for x: $\log_2(3x-1) = 3$

(a) $x = \frac{7}{3}$

(b) $x = \frac{10}{3}$

(c) $x = \frac{8}{3}$

(d) $x = \frac{4}{3}$

(e) None of these

6.6 C. Solve for x: $2^{x+1} = 6$

(a) $x = \ln 3$

(b) $x = \ln 4$

(c) $x = \ln 4 - \ln 2$

(d) $x = \dfrac{\ln 3}{\ln 2}$

(e) None of these

6.6 D. Solve for x: $2^{2x+1} = \left(\frac{1}{2}\right)^x$

(a) $x = \frac{1}{3}$

(b) $x = 0$

(c) $x = -\frac{1}{3}$

(d) $x = -1$

(e) None of these

6.6 E. Solve for x: $5^x = 3^{1-2x}$

(a) $x = 3/7$

(b) $x = 11/7$

(c) $\dfrac{\ln 3}{\ln 5 + 2\ln 3}$

(d) $\dfrac{1}{\ln 5 + 2\ln 3}$

(e) None of these

Answer Key

6.1 #11a. (c)

6.1 #11. (a)

6.1 #15a. (b)

6.1 A. (d)

6.1 #31b. (d)

6.1 Example 4a. (a)

6.1 #35a1. (d)

6.1 #35a2. (e)

6.1 #36. (d)

6.1 #61. (d)

6.1 #63a. (c)

6.1 B. (a)

6.2 A. (d)

6.2 #49. (c)

6.2 B. (a)

6.2 #57a. (b)

6.2 #57b. (c)

6.2 #59a. (c)

6.2 #59b. (a)

6.2 #60. (a)

6.2 #63a. (c)

6.2 #63b. (c)

6.2 C. (d)

6.2 #94. (d)

6.3 A. (d)

6.3 B. (d)

6.3 C. (c)

6.3 #43. (c)

6.3 #52a. (a)

6.3 #52b. (e)

6.3 #55. (e)

6.3 D. (d)

6.3 #61. (d)

6.3 #63. (d)

6.3 E. (c)

6.3 #85. (c)

6.3 #86. (e)

6.4 A. (b)

6.4 B. (e)

6.4 C. (a)

6.4 D. (c)

6.4 E. (b)

6.4 #29. (e)

6.4 #33. (e)

6.4 #35. (b)

6.4 F. (d)

6.4 #43. (d)

6.4 #82. (a)

6.4 G. (b)

6.4 #89. (e)

6.5 #14. (a)

6.5 A. (b)

6.5 B. (c)

6.5 #29. (b)

6.5 #46. (a)

6.5 #49. (c)

6.5 #72. (c)

6.5 #83. (b)

6.5 #85. (b)

6.5 #87. (c)

6.6 A. (d)

6.6 B. (e)

6.6 C. (d)

6.6 D. (c)

6.6 E. (c)

Solutions

6.1 A. Let $f(x) = 2x^2 + 1$ and let $g(x) = x + 3$. Find the composite function $(f \circ g)(x)$.

Solution: (d) Write the formula for $f(x)$ in this way: $f(\quad) = 2(\quad)^2 + 1$. Then you have $(f \circ g)(x) = f(g(x)) = 2g(x)^2 + 1 = 2(x+3)^2 + 1 = 2(x^2+6x+9)+1 = 2x^2+12x+19$.

6.1 #36. Find the domain of $f \circ g$ if $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{-2}{x}$.

Solution: (d) $\{x \mid x \neq 0 \text{ and } x \neq 2/3\}$ First, you must exclude $x = 0$, since it is not in the domain of the first function $g(x)$. If you compute the composite function $f(g(x))$, you get $f(g(x)) = \dfrac{1}{-\frac{2}{x} + 3} = \frac{x}{-2+3x}$, so you must *also* exclude $x = 2/3$ (found by setting the denominator $-2 + 3x$ equal to 0).

6.1 B. If $f(x) = 3x^2 - 7$ and $g(x) = 2x + a$, find a so that the graph of $f \circ g$ crosses the y-axis at 5.

Solution: (a) One method of solution is to compute the composite function $f(g(x))$. You get $f(g(x)) = 3(2x + a)^2 - 7 = 3(4x^2 + 4ax + a^2) - 7 = 12x^2 + 12ax + 3a^2 - 7$. The problem asks you to find a so that the y-intercept is 5. Since the y-intercept is $3a^2 - 7$, you need to solve $3a^2 - 7 = 5$. You get $3a^2 = 12$, so $a = \pm 2$.

Another method of solution is to find the y-intercept in two steps. The first is by substituting $x = 0$ into $g(x)$. You get $g(0) = a$, and then, as the second step, you get $f(g(0)) = 3a^2 - 7$. The answer $a = \pm 2$ again comes from the solution of the equation $3a^2 - 7 = 5$.

6.2 A. If $f(x)$ has an inverse, and $(2, -\frac{1}{2})$ is on the graph of $f(x)$, then what point is on the graph of $f^{-1}(x)$?

Solution: (d) $(-\frac{1}{2}, 2)$ If (a, b) is on the graph of $f(x)$, then (b, a) is on the graph of $f^{-1}(x)$.

6.2 B. The function $f(x) = \sqrt{x - 2}$, for $x \geq 2$, is a one-to-one function. Find the inverse function f^{-1}.

Solution: (a) $f^{-1}(x) = x^2 + 2$, for $x \geq 0$.
Step 1. Write the function in the form $y = \sqrt{x - 2}$.
Step 2. Interchange x and y to get $x = \sqrt{y - 2}$.
Step 3. Solve for y in terms of x. $x = \sqrt{y - 2}$ $x^2 = (\sqrt{y - 2})^2 = y - 2$ $y = x^2 + 2$

To find the domain of $f^{-1}(x)$, it may be easiest to find the range of $f(x)$. Since $x \geq 2$, this includes the square root of every number ≥ 0, so the range of $f(x)$ is $\{y \mid y \geq 0\}$. This means that the domain of $f^{-1}(x)$ is $\{x \mid x \geq 0\}$, so the solution is the one given above.

6.2 #60. The function $f(x) = \frac{4}{2-x}$, for $x \neq 2$, is a one-to-one function. Find the inverse function f^{-1}.

Solution: (a) $f^{-1}(x) = 2 - \frac{4}{x}$ Write $y = \frac{4}{2-x}$, then exchange x and y and solve for y. $x = \frac{4}{2-y}$ $(2-y)x = 4$ $2x - yx = 4$ $-yx = 4 - 2x$ $y = \frac{4-2x}{-x}$ $y = \frac{2x-4}{x} = 2 - \frac{4}{x}$

6.2 C. The function $f(x) = 3x - 2$ is a one-to-one function. Find the inverse function f^{-1}.
Solution: (d) $f^{-1}(x) = \frac{1}{3}x + \frac{2}{3}$ $y = 3x - 2$ $x = 3y - 2$ $x + 2 = 3y$ $y = \frac{1}{3}x + \frac{2}{3}$

6.2 #94. The period T of a simple pendulum is $T = 2\pi\sqrt{\frac{x}{g}}$, where x is its length and g is a constant (the acceleration due to gravity). Solve for x as a function of T.

Solution: (d) $T = 2\pi\sqrt{\frac{x}{g}}$, $T^2 = 4\pi^2\left(\sqrt{\frac{x}{g}}\right)^2$ $T^2 = 4\pi^2\frac{x}{g}$ $gT^2 = 4\pi^2 x$

Answer: $x = \frac{gT^2}{4\pi^2}$

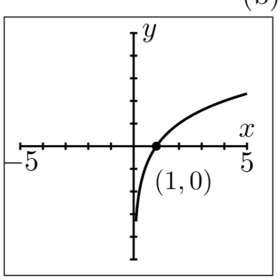
6.3 A. Which answer describes the graph of the exponential function $f(x) = e^x$?

(a) The graph goes through $(0, e)$ and decreases as x increases.
(b) The graph goes through $(0, e)$ and increases as x increases.
(c) The graph goes through $(0, 1)$ and decreases as x increases.
(d) The graph goes through $(0, 1)$ and increases as x increases.
(e) The graph is a straight line through $(1, e)$.

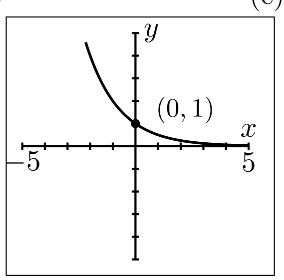
Solution: (d) When $x = 0$, we have $f(0) = e^0 = 1$, so the answer must be (c) or (d). As the exponent x increases, the values of e^x get larger and larger, so the y-values increase as x increases, and the answer must be (d).

6.3 B. Which of the following is the graph of $y = 2^x$?

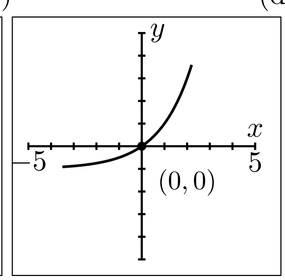
(a)



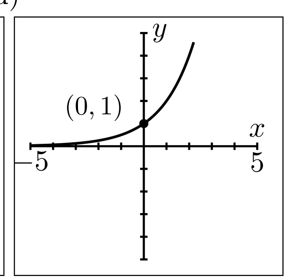
(b)



(c)



(d)



Solution: (d) The graph must go through $(0, 1)$, and must increase as x increases.

6.3 C. Which exponential function is represented by this graph? (see the original problem)

Solution: The first choice (a) $f(x) = -2x^2 + x$ is not an exponential function. The graph is decreasing, not increasing, so it cannot be (d) $f(x) = 1 + 2^x$ or (e) $f(x) = 1 + e^x$. We need to decide between (b) $f(x) = 1 - 2^{-x}$ and (c) $f(x) = 1 - 2^x$. Note that $(0, 0)$ and $(1, -1)$ are on the graph. Both (b) and (c) have $f(0) = 0$, but only (c) has $f(1) = -1$.

6.4 A. Which answer describes the graph of the logarithmic function $f(x) = \ln x$?

(a) The graph goes through $(0, 1)$ and has $x = 0$ as a vertical asymptote.
(b) The graph goes through $(1, 0)$ and has $x = 0$ as a vertical asymptote.
(c) The graph goes through $(0, 1)$ and has $y = 0$ as a horizontal asymptote.
(d) The graph goes through $(1, 0)$ and has $y = 0$ as a horizontal asymptote.
(e) The graph is a straight line through $(0, 1)$ and $(e, 1)$.

Solution: (b) The log function is the inverse of the exponential function, whose graph goes through $(0, 1)$, so its graph must go $(1, 0)$. It has a vertical asymptote, but no horizontal asymptote.

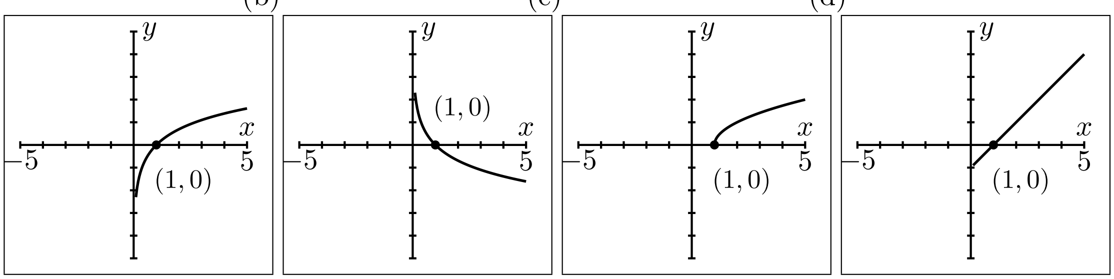
6.4 B. List the properties of the graph of $y = \ln x$.

A: The graph has a vertical asymptote at $x = 1$.
B: The graph has a vertical asymptote at $x = 0$.
C: The graph goes through $(e, 0)$.
D: The graph goes through $(1, 0)$.
E: The graph has a horizontal asymptote.
F: The graph increases as x increases.

Solution: (e) B, D, and F The graph does have a vertical asymptote, the y-axis, so B holds. Since $\ln(e) = 1$, condition C is false. The graph goes through $(1, 0)$ so D is true. The graph has no horizontal asyptote so E is false. Finally, F is true.

6.4 C. Which of the following is the graph of $y = \ln x$?

(a) (b) (c) (d)



Solution: (a) Use the properties of $y = \ln x$. The graph has a vertical asymptote at $x = 0$, goes through $(1, 0)$, and increases as x increases.

6.4 D. Which of the following pairs of functions are inverses of each other?

(a) $\ln(x)$ and 10^x (b) $\log(x)$ and e^x (c) $\ln(x)$ and e^x
(d) $\log_{2.7}(x)$ and e^x (e) $\log_2(x)$ and $\left(\frac{1}{2}\right)^x$

Solution: (c) The base must be the same in both functions. Since $\ln(x) = \log_e(x)$, the only pair for which this is true is (c).

6.4 E. If $f(x) = \log_3(x)$, what is $f^{-1}(x)$?

Solution: (b) $f^{-1}(x) = 3^x$ The inverse is an exponential function with the same base.

6.4 F. The domain of $f(x) = \log(1 - 5x)$ is

Solution: (d) $(-\infty, \frac{1}{5})$ The log function is only defined for positive values, so the domain is found by setting $1 - 5x > 0$. We get $x < \frac{1}{5}$.

6.4 #82. Find the vertical asymptote of the graph of $f(x) = 2 - \log_3(x+1)$.
Solution: (a) $x = -1$ The domain of $f(x)$ is $\{x \mid x+1 > 0\} = \{x \mid x > -1\}$.

6.4 G. The equation $\log_\pi x = \frac{1}{2}$ can be written in exponential form as
Solution: (b) To remove the $\log_\pi$, substitute both sides of the equation into the inverse function $g(x) = \pi^x$ of $f(x) = \log_\pi(x)$. This works because $\pi^{\log_\pi(x)} = x$ for all $x > 0$.
$\pi^{\log_\pi(x)} = \pi^{1/2}$ $x = \pi^{1/2}$

6.5 A. $(\log_2 6)(\log_6 8) =$

Solution: (b) Convert $\log_6 8$ to base 2 using the formula $\log_a M = \dfrac{\log_b M}{\log_b a}$ (see p455). This gives $(\log_2 6)(\log_6 8) = (\log_2 6)\left(\dfrac{\log_2 8}{\log_2 6}\right) = \dfrac{(\log_2 6)(\log_2 8)}{\log_2 6} = \log_2 8 = 3.$

6.5 B. $(\log_3 6)(\log_6 9) =$

Solution: (c) $(\log_3 6)(\log_6 9) = (\log_3 6)\left(\dfrac{\log_3 9}{\log_3 6}\right) = \log_3 9 = 2.$

6.5 #46. $\log\left(\dfrac{x^3\sqrt{x+1}}{(x-2)^2}\right) =$

Solution: (a)

$$\begin{aligned}\log\left(\frac{x^3\sqrt{x+1}}{(x-2)^2}\right) &= \log(x^3) + \log(\sqrt{x+1}) - \log((x-2)^2) \\ &= 3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)\end{aligned}$$

6.5 #72. $\log_\pi \sqrt{2} =$

Solution: (c) $\log_\pi \sqrt{2} = \dfrac{\ln\sqrt{2}}{\ln \pi} = \dfrac{\frac{1}{2}\ln 2}{\ln \pi} = \dfrac{\ln 2}{2\ln \pi}$ by formula (9) on p455.

6.6 A. Solve for x: $\ln(x+1) + \ln(x) = \ln(6)$
Solution: (d) $\ln(x+1) + \ln(x) = \ln(6)$ $\ln(x+1)(x) = \ln(6)$ $x^2 + x = 6$
$x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3$ or $x = 2$
Since $\ln x$ is defined only for positive numbers, $x = -3$ cannot be a solution, and the correct answer is $x = 2$.

6.6 B. Solve for x: $\log_2(3x-1) = 3$
Solution: (e) To simplify by removing the term $\log_2$, use the inverse function 2^x.
$\log_2(3x-1) = 3$ $2^{\log_2(3x-1)} = 2^3$ $3x - 1 = 8$ $3x = 9$ $x = 3$

6.6 C. Solve for x: $2^{x+1} = 6$

Solution: (d) Since the answers are given in terms of ln, take the natural log of both sides. $2^{x+1} = 6$ $\ln(2^{x+1}) = \ln 6$ $(x+1)\ln 2 = \ln 6$ $(\ln 2)x + \ln 2 = \ln 6$
$(\ln 2)x = \ln 6 - \ln 2 = \ln \frac{6}{2} = \ln 3$ $x = \dfrac{\ln 3}{\ln 2}$

6.6 D. Solve for x: $2^{2x+1} = \left(\frac{1}{2}\right)^x$

Solution: (c) Rewrite the problem so that the base is the same on both sides.
$2^{2x+1} = \left(\frac{1}{2}\right)^x$ $2^{2x+1} = \left(2^{-1}\right)^x = 2^{-x}$ Now you can equate the exponents since both sides have base 2. $2x + 1 = -x$ $x = -\frac{1}{3}$

6.6 E. Solve for x: $5^x = 3^{1-2x}$

Solution: (c) Take the natural log of both sides.

$5^x = 3^{1-2x}$ $\ln(5^x) = \ln(3^{1-2x})$ $x \ln 5 = (1 - 2x) \ln 3$

$x \ln 5 + 2x \ln 3 = \ln 3$ $(\ln 5 + 2 \ln 3)x = \ln 3$ $x = \frac{\ln 3}{\ln 5 + 2 \ln 3}$