Chapter 0

Review

Section summaries

Section R.1: Real Numbers

You should review the rules for working with numbers, especially those for fractions (see page 13):

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \qquad \text{where } b, c, d \text{ are nonzero}$$

Review problems: p16 #71,73,77,81,83

Section R.2: Algebra Essentials

In this section the rules for exponents are particularly important.

$$a^m a^n = a^{m+n} \qquad a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad \left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

Explanation: The shorthand notation that we use in writing a^3 instead of aaa dates back only to the 1600's, so it is a fairly recent invention (in the long history of math). When we multiply powers in this form, we can expand the powers and just count how many terms we have in the result. For example, $a^4 \cdot a^3 = aaaa \cdot aaa = a^7$, This method works for all positive exponents and leads to the general rule $a^m \cdot a^n = a^{m+n}$.

It's very useful to allow the exponents to be 0 or a negative number. But then there's no way to interpret the shorthand notation as a repeated product of a. We should at least make sure that any definition we give is consistent with the general formula $a^m \cdot a^n = a^{m+n}$.

To define a^0, we note that by the general rule for exponents we should have $a^m \cdot a^0 = a^{m+0} = a^m$. Dividing both sides of the equation by a^m gives us $\frac{a^m \cdot a^0}{a^m} = \frac{a^m}{a^m}$, and so we should have $a^0 = 1$. This shows that to be consistent with the rule for exponents we must define $a^0 = 1$.

If n is a positive integer, then to define a^{-n} we can give a similar argument. We should have $a^n \cdot a^{-n} = a^{n-n} = a^0 = 1$. Dividing both sides of the equation by a^n shows that we must define $a^{-n} = \dfrac{1}{a^n}$.

Review problems: p27 #71,87,91

Section R.3: Geometry Essentials

You need to know the **Pythagorean theorem**: $a^2 + b^2 = c^2$ for any right triangle with legs a and b and hypotenuse c.

Area of a rectangle = length $\cdot$ width;

area of a triangle = $\dfrac{1}{2}$ base $\cdot$ height;

area of a circle = $\pi \cdot (\text{radius})^2$.

Volume of a rectangular box = length $\cdot$ width $\cdot$ height.

Circumference of a circle = $\pi \cdot$ diameter.

Review problems: p36 #21,27,31,47,49

Section R.4: Polynomials

A **polynomial** is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$. If a_n is nonzero, then it is the **leading coefficient** and the polynomial has **degree** n. Polynomials are added by adding like terms, and multiplied by using the distributive laws. (A special case is to use FOIL to multiply $(ax + b)(cx + d)$.)

You need to remember that $(x+a)^2 = x^2+2ax+a^2$, and $(x+a)^3 = x^3+3ax^2+3a^2x+a^3$.

See Example 14 on page 46 for a good example of long division of polynomials.

Review problems: p48 #79,83,87,93,99,103

Section R.5: Factoring Polynomials

In this section there are a few formulas to remember. The factorization of the difference of two squares is given by $x^2 - a^2 = (x - a)(x + a)$. We also have $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$ and $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$, but $x^2 + a^2$ cannot be factored (using real numbers). The polynomials $x^2 + 2ax + a^2 = (x + a)^2$ and $x^2 - 2ax + a^2 = (x - a)^2$ are perfect squares.

Review problems: p56 #77,87,93,105,121

Section R.7: Rational Expressions

The quotient of two polynomials is called a **rational expression**. The rules for working with them are the same as for working with fractions (rational numbers).

$$\frac{a(x)}{b(x)} \cdot \frac{c(x)}{d(x)} = \frac{a(x)c(x)}{b(x)d(x)} \qquad \frac{a(x)}{b(x)} \div \frac{c(x)}{d(x)} = \frac{a(x)}{b(x)} \cdot \frac{d(x)}{c(x)} = \frac{a(x)d(x)}{b(x)c(x)}$$

$$\frac{a(x)}{b(x)} + \frac{c(x)}{d(x)} = \frac{a(x)d(x) + b(x)c(x)}{b(x)d(x)}$$

When you add or subtract rational expressions, it often turns out that using the product of their denominators makes the fraction more complicated than necessary. The least common multiple method (see page 65) can save a lot of work.

Review problems: p70 #45,71,81,85

Section R.8: Roots; Rational Exponents

The number b is the nth root of a, written $b = \sqrt[n]{a}$, if $b^n = a$. This can also be written as $b = a^{1/n}$. See page 73 for some properties of radicals.

Rational exponents (also called fractional exponents) can be evaluated using roots and powers:

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m \quad \text{or} \quad a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}.$$

Review problems: p78 #47,53,71,81,89,91

Sample Questions

R.2 #92. After simplifying, the **numerator** of $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y}$ is

(a) $2x^2$
(b) $2x^2z$
(c) yz
(d) 1
(e) 4

R.3 #15. The lengths of the legs of a right triangle are 7 and 24. Find the length of the hypotenuse.

(a) 84 (b) 31 (c) 25 (d) $\sqrt{97}$ (e) $\sqrt{62}$

R.3 A. Find the **volume** V and the **surface area** S of a rectangular box with length 4 feet, width 2 feet, and height 5 feet.

(a) $V = 40$ cubic feet and $S = 76$ square feet
(b) $V = 40$ cubic feet and $S = 11$ square feet
(c) $V = 20$ cubic feet and $S = 76$ square feet
(d) $V = 20$ cubic feet and $S = 11$ square feet
(e) None of these

R.3 #31. Find the **volume** V and **surface area** S of a rectangular box with length 8 feet, width 4 feet, and height 7 feet.

(a) $V = 214$ cubic feet and $S = 116$ square feet
(b) $V = 214$ cubic feet and $S = 232$ square feet
(c) $V = 224$ cubic feet and $S = 116$ square feet
(d) $V = 224$ cubic feet and $S = 232$ square feet
(e) None of these

R.4 A. Perform the indicated operation and simplify: $(2x-5)(3x+4)$

(a) $6x^2 - 20$
(b) $6x^2 - 7x - 20$
(c) $5x^2 - 7x - 20$
(d) $5x^2 - 2x - 20$
(e) $5x - 1$

R.4 B. Perform the indicated operation and simplify: $(2x^2)^3(4x^3)$

(a) $32x^9$
(b) $32x^8$
(c) $8x^8$
(d) $6x^8$
(e) None of these

R.4 C. Simplify: $(x^2 - 3x + 1) - (2x - 5)$

(a) $-2x^3 + 11x^2 - 17x + 5$
(b) $x^2 - x - 4$
(c) $x^2 - 5x - 4$
(d) $x^2 - 5x + 6$
(e) $x^2 - x + 6$

R.4 Example 9b. Expand and simplify: $(x - 1)^3$

(a) $x^3 - 1$
(b) $x^3 - x^2 - x + 1$
(c) $x^3 - 3x^2 - x + 1$
(d) $x^3 - 3x^2 + 3x - 1$
(e) None of these

R.4 #64. Multiply and simplify: $(x - 3y)(-2x + y)$

(a) $-2x^2 - 5xy - 3y^2$
(b) $-2x^2 - 5xy + 3y^2$
(c) $-2x^2 - 7xy - 3y^2$
(d) $-2x^2 - 7xy + 3y^2$
(e) None of these

R.4 #84. Expand and simplify: $(2x + 3y)^2$

(a) $2x^2 + 3y^2$
(b) $4x^2 + 9y^2$
(c) $4x^2 + 12xy + 9y^2$
(d) $4x^2 + 6xy + 9y^2$
(e) None of these

R.4 #87. Expand and simplify: $(2x + 1)^3$

(a) $8(x^3 + 1)$
(b) $8x^3 + 1$
(c) $8x^3 + 4x^2 + 2x + 1$
(d) $8x^3 + 12x^2 + 6x + 1$
(e) None of these

R.4 D. What is the **remainder** when $x^2 + x + 1$ is divided by $x - 2$?

(a) 7
(b) 6
(c) 5
(d) 4
(e) None of these

R.4 #93. When $5x^4 - 3x^2 + x + 1$ is divided by $x^2 + 2$ the **remainder** is

(a) $x + 27$

(b) $x + 25$

(c) $x + 15$

(d) $x + 13$

(e) None of these

R.4 #97. When $2x^4 - 3x^3 + x + 1$ is divided by $2x^2 + x + 1$ the **quotient** is

(a) $x^2 - 2x$

(b) $x^2 + 2x$

(c) $x^2 - 2x + \frac{1}{2}$

(d) $x^2 + 2x - \frac{1}{2}$

(e) None of these

R.4 #103. When $x^3 - a^3$ is divided by $x - a$ the **quotient** is

(a) $x^2 + ax + a^2$

(b) $x^2 - ax + a^2$

(c) $x^2 + a^2$

(d) $x^2 - a^2$

(e) None of these

R.5 Example 11. Factor the following expression completely: $x^2 - x - 12$

(a) $(x - 6)(x + 2)$

(b) $(x + 4)(x - 3)$

(c) $(x + 6)(x - 2)$

(d) $(x - 4)(x + 3)$

(e) None of these

R.5 A. Which one of the following is a factor of $4x^2 + 5x - 6$?

(a) $4x + 3$

(b) $x - 2$

(c) $4x - 3$

(d) $2x - 3$

(e) $4x + 5$

R.5 B. Factor completely: $ac^2 + 5bc^2 - a - 5b$

(a) $(a + 5b)(c + 1)(c - 1)$

(b) $(a - 5b)(c + 1)(c - 1)$

(c) $(a + 5b)(c + 1)^2$

(d) $(a - 5b)(c + 1)^2$

(e) None of these

R.5 #111. Factor completely: $3(x^2 + 10x + 25) - 4(x + 5)$

(a) $3x^2 + 26x + 95$

(b) $(x + 5)(3x - 19)$

(c) $(x + 5)(3x + 11)$

(d) $-(x + 5)^2$

(e) None of these

R.5 #122. Factor completely: $4(x + 5)^3(x - 1)^2 + 2(x + 5)^4(x - 1)$

(a) $8(x + 5)^7(x - 1)^3$

(b) $2(x + 5)^3(x - 1)$

(c) $6(x + 5)^3(x - 1)(x + 1)$

(d) $2(x + 5)^3(x - 1)(3x + 7)$

(e) None of these

R.7 A. Reduce the following rational expression to lowest terms: $\dfrac{x^2 + x - 6}{x^2 - 4}$

(a) $\dfrac{x + 3}{x + 2}$

(b) $\dfrac{x + 3}{x - 2}$

(c) $\dfrac{x - 6}{-4}$

(d) $x - 2$

(e) None of these

R.7 B. Simplify; leave your answer in factored form: $\dfrac{1}{x - 1} - \dfrac{2}{x + 2}$

(a) $\dfrac{-1}{2x + 1}$

(b) $\dfrac{2x + 1}{(x + 1)(x + 2)}$

(c) -1

(d) $\dfrac{-x + 4}{(x - 1)(x + 2)}$

(e) None of these

R.7 #45. After simplifying, the **numerator** of $\dfrac{4}{x - 1} - \dfrac{2}{x + 2}$ is

(a) 2

(b) -8

(c) $2(x + 3)$

(d) $2(x - 4)$

(e) None of these

R.7 #47. After simplifying, the **numerator** of $\dfrac{x}{x+1} + \dfrac{2x-3}{x-1}$ is

(a) $3(x-1)$

(b) $3x^2 - 2x + 3$

(c) $3x^2 - 4x + 3$

(d) $3x^2 + 4x + 3$

(e) None of these

R.7 Example 10. Simplify (assuming $x \neq -3, 0$): $\dfrac{\dfrac{1}{2} + \dfrac{3}{x}}{\dfrac{x+3}{4}}$

(a) $\dfrac{2(x+2)}{x^2}$

(b) $\dfrac{8}{x(x+3)}$

(c) $\dfrac{(x+6)(x+3)}{8x}$

(d) $\dfrac{2(x+6)}{x(x+3)}$

(e) None of these

R.7 C. Simplify: $\dfrac{\left(\dfrac{x}{x+1}\right)}{\left(\dfrac{2x+2}{x^2}\right)}$

(a) $\dfrac{x^2}{2(x+2)}$

(b) $\dfrac{x^3}{2(x+1)^2}$

(c) $\dfrac{x^3}{2(x^2+1)}$

(d) $\dfrac{2}{x}$

(e) None of these

R.7 #66. After simplifying $\dfrac{2}{(x+2)^2(x-1)} - \dfrac{6}{(x+2)(x-1)^2}$, the **numerator** is

(a) $-2(4x+7)$

(b) $-2(4x-5)$

(c) $-2(2x-5)$

(d) $-2(2x+7)$

(e) $-2(2x-7)$

R.7 #72. After simplifying, the **numerator** of $\frac{1}{h}\left[\frac{1}{(x+h)^2}-\frac{1}{x^2}\right]$ is

(a) $-2x-h$

(b) $-2x+h$

(c) $2x-h$

(d) $2x+h$

(e) None of these

R.7 #76. After simplifying, the **denominator** of $\dfrac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}}$ is

(a) $x+1$

(b) $(x+1)^2$

(c) x^2-1

(d) $(x-1)^2$

(e) None of these

R.7 D. Simplify and factor: $\dfrac{4+\frac{1}{x^2}}{25-\frac{1}{x^2}}$.

(a) $\dfrac{4}{25}$

(b) $\dfrac{4x^2+1}{(x+5)(x-5)}$

(c) $\dfrac{(2x+1)(2x-1)}{(5x+1)(5x-1)}$

(d) $\dfrac{(x+2)(x-2)}{(x+5)(x-5)}$

(e) $\dfrac{4x^2+1}{(5x+1)(5x-1)}$

R.8 A. Simplify: $2\sqrt{3}-\sqrt{48}$

(a) $2\sqrt{3}$

(b) $-14\sqrt{3}$

(c) $-2\sqrt{3}$

(d) $3\sqrt{5}$

(e) -6

R.8 B. Simplify: $2\sqrt{3}+2\sqrt{12}$

(a) $2\sqrt{15}$

(b) $6\sqrt{3}$

(c) $10\sqrt{3}$

(d) 30

(e) None of these

R.8 Example 8a. Simplify: $(x^{2/3}y)(x^{-2}y)^{1/2}$

(a) $x^{8/3}$

(b) $\dfrac{y}{x^{2/3}}$

(c) $\dfrac{y^{3/2}}{x^{4/3}}$

(d) $\dfrac{y^{3/2}}{x^{1/3}}$

(e) None of these

R.8 Example 8c. Simplify: $\left(\dfrac{9x^2y^{1/3}}{x^{1/3}y}\right)^{1/2}$

(a) $3x$

(b) $\dfrac{3x^{9/5}}{y^{1/3}}$

(c) $\dfrac{3x^{5/6}}{y^{1/3}}$

(d) $\dfrac{9x^{9/5}}{y^{1/3}}$

(e) None of these

R.8 #18. Simplify (assuming that all variables are positive: $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}}$

(a) $\dfrac{1}{\sqrt{3x}}$

(b) $\dfrac{\sqrt{3}}{x}$

(c) $\dfrac{x}{\sqrt{3}}$

(d) $\dfrac{1}{3\sqrt{x}}$

(e) None of these

R.8 #30. Simplify: $2\sqrt{12} - 3\sqrt{27}$

(a) $-\sqrt{15}$

(b) $-19\sqrt{3}$

(c) $-5\sqrt{3}$

(d) $-6\sqrt{324}$

(e) None of these

R.8 C. Simplify: $4\sqrt[3]{7} - 3\sqrt[3]{56}$

(a) 2

(b) $-3\sqrt[3]{49}$

(c) $-4\sqrt[3]{7}$

(d) $2\sqrt[3]{7}$

(e) None of these

R.8 D. Rationalize the denominator: $\dfrac{10}{4-\sqrt{2}}$

(a) $5\sqrt{2}$

(b) $\dfrac{5(4-\sqrt{2})}{7}$

(c) $\dfrac{5(4+\sqrt{2})}{7}$

(d) $\dfrac{5(4-\sqrt{2})}{9}$

(e) $\dfrac{5(4+\sqrt{2})}{9}$

R.8 E. Simplify $\left(\dfrac{1}{64}\right)^{-2/3}$

(a) 16

(b) 512

(c) $-\frac{1}{16}$

(d) $-\frac{1}{96}$

(e) None of these

R.8 F. Simplify $\left(\dfrac{27}{8}\right)^{-2/3}$

(a) $\frac{9}{4}$

(b) $\frac{4}{9}$

(c) $\frac{-4}{9}$

(d) $\frac{-16}{81}$

(e) None of these

R.8 G. Factor the expression $x^{1/2}(x^2+x)+x^{3/2}-24x^{1/2}$ (where $x \geq 0$).

(a) $x^{1/2}(x+1)(x-3)$

(b) $x^{3/2}(x+1)(x-3)$

(c) $x^{3/2}(x+2)(x-6)$

(d) $x^{1/2}(x+6)(x-4)$

(e) $(x+6)(x-4)$

R.8 H. Multiply and simplify $(2\sqrt{x}-3)(2\sqrt{x}+5)$

(a) $4x+4\sqrt{x}-15$

(b) $2x+4\sqrt{x}-15$

(c) $4x-15$

(d) $4\sqrt{x}-15$

(e) None of these

R.8 #37. Simplify: $\sqrt[3]{16x^4} - \sqrt[3]{2x}$

(a) $\sqrt[3]{14}x$

(b) $(8x^3 - 1)\sqrt[3]{2x}$

(c) $(8x - 1)\sqrt[3]{2x}$

(d) $(2x - 1)\sqrt[3]{2x}$

(e) None of these

R.8 #53. After rationalizing the denominator of $\dfrac{\sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$, the numerator is

(a) $2x + h$

(b) $2x + h - 2\sqrt{x(x+h)}$

(c) $2x + h + 2\sqrt{x(x+h)}$

(d) $2x - 2\sqrt{x(x+h)}$

(e) None of these

Answer Key

R.2 #92. (d)

R.3 #15. (c)

R.3 A. (a)

R.3 #31. (d)

R.4 A. (b)

R.4 B. (a)

R.4 C. (d)

R.4 Example 9b. (d)

R.4 #64. (e)

R.4 #84. (c)

R.4 #87. (d)

R.4 D. (a)

R.4 #93. (a)

R.4 #97. (c)

R.4 #103. (a)

R.5 Example 11. (d)

R.5 A. (c)

R.5 B. (a)

R.5 #111. (c)

R.5 #122. (c)

R.7 A. (a)

R.7 B. (d)

R.7 #45. (e)

R.7 #47. (e)

R.7 Example 10. (d)

R.7 C. (b)

R.7 #66. (d)

R.7 #72. (a)

R.7 #76. (b)

R.7 D. (e)

R.8 A. (c)

R.8 B. (b)

R.8 Example 8a. (d)

R.8 Example 8c. (c)

R.8 #18. (e)

R.8 #30. (c)

R.8 C. (e)

R.8 D. (c)

R.8 E. (a)

R.8 F. (b)

R.8 G. (d)

R.8 H. (a)

R.8 #37. (d)

R.8 #53. (b)

Solutions

R.2 #92. $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y} = \dfrac{4}{8x^2x^4y(yz)} = \dfrac{1}{2x^6y^2z}$

R.3 A. Find the **volume** V and the **surface area** S of a rectangular box with length 4 feet, width 2 feet, and height 5 feet.

Solution: The volume is length times width times height, so we get $4 \cdot 2 \cdot 5 = 40$ cubic feet. To find the surface area: there are 4 sides, two that are 4×5 and two that are 2×5. The total area for the sides is $20 + 20 + 10 + 10 = 60$ square feet. The top and bottom of the box are 4×2, adding $8 + 8 = 16$ more square feet. The total surface area is 76 square feet. The answer is $V = 40$ cubic feet and $S = 80$ square feet.

R.4 A. $(2x - 5)(3x + 4) = (2x)(3x + 4) - (5)(3x + 4) = 6x^2 + 8x - 15x - 20 = 6x^2 - 7x - 20$

R.4 B. $(2x^2)^3(4x^3) = 2^3(x^2)^3(4x^3) = 8x^6 \cdot 4x^3 = (8 \cdot 4)(x^6x^3) = 32x^9$

R.4 C. $(x^2 - 3x + 1) - (2x - 5) = x^2 - 3x + 1 - 2x + 5 = x^2 + (-3x - 2x) + (1 + 5) = x^2 - 5x + 6$

R.4 #64. $(x - 3y)(-2x + y) = x(-2x + y) + (-3y)(-2x + y) = -2x^2 + xy + 6xy - 3y^2 = -2x^2 + 7xy - 3y^2$

R.4 #84. $(2x + 3y)^2 = (2x + 3y)(2x + 3y) = 4x^2 + 6xy + 6xy + 9y^2 = 4x^2 + 12xy + 9y^2$

R.4 D. What is the **remainder** when x^2+x+1 is divided by $x-2$?

Solution: The remainder is +7.

$$
\begin{array}{rr|rrr}
 & & & x & +3 \\
\cline{3-5}
x & -2 & x^2 & +x & +1 \\
 & & x^2 & -2x & \\
\cline{3-4}
 & & & +3x & +1 \\
 & & & +3x & -6 \\
\cline{4-5}
 & & & & +7
\end{array}
$$

R.5 A. $4x^2+5x-6=(4x-3)(x+2)$

R.5 B. $ac^2+5bc^2-a-5b=c^2(a+5b)-a-5b=c^2(a+5b)+(-1)(a+5b)=(c^2-1)(a+5b)=$ $(c+1)(c-1)(a+5b)$

R.5 #122. $4(x+5)^3(x-1)^2+2(x+5)^4(x-1)=$
$(2\cdot 2)(x+5)^3(x-1)(x-1)+2(x+5)^3(x+5)(x-1)=$ (find common terms)
$2(x+5)^3(x-1)\cdot(2(x-1)+(x+5))=$ (factor out common terms)
$2(x+5)^3(x-1)(2x-2+x+5)=$
$2(x+5)^3(x-1)(3x+3)=$
$6(x+5)^3(x-1)(x+1)$

R.7 A. $\dfrac{x^2+x-6}{x^2-4}=\dfrac{(x+3)(x-2)}{(x+2)(x-2)}=\dfrac{x+3}{x+2}$

R.7 B. $\dfrac{1}{x-1}-\dfrac{2}{x+2}=\dfrac{x+2}{(x-1)(x+2)}-\dfrac{2(x-1)}{(x-1)(x+2)}=\dfrac{x+2-2x+2}{(x-1)(x+2)}=\dfrac{-x+4}{(x-1)(x+2)}$

R.7 C. $\dfrac{\left(\dfrac{x}{x+1}\right)}{\left(\dfrac{2x+2}{x^2}\right)}=\left(\dfrac{x}{x+1}\right)\left(\dfrac{x^2}{2(x+1)}\right)=\dfrac{x^3}{2(x+1)^2}$

R.7 #66. $\dfrac{2}{(x+2)^2(x-1)}-\dfrac{6}{(x+2)(x-1)^2}=\dfrac{2(x-1)}{(x+2)^2(x-1)^2}-\dfrac{6(x+2)}{(x+2)^2(x-1)^2}=$

$\dfrac{2x-2-6(x+2)}{(x+2)^2(x-1)^2}=\dfrac{2x-2-6x-12}{(x+2)^2(x-1)^2}=\dfrac{-4x-14}{(x+2)^2(x-1)^2}=\dfrac{-2(2x+7)}{(x+2)^2(x-1)^2}$

R.7 #72. $\dfrac{1}{h}\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]=\dfrac{1}{h}\left[\dfrac{x^2-(x+h)^2}{(x+h)^2x^2}\right]=\dfrac{1}{h}\left[\dfrac{x^2-(x^2+2xh+h^2)}{(x+h)^2x^2}\right]=$

$\dfrac{1}{h}\left[\dfrac{x^2-x^2-2xh-h^2}{(x+h)^2x^2}\right]=\dfrac{1}{h}\left[\dfrac{-2xh-h^2}{(x+h)^2x^2}\right]=\dfrac{1}{h}\left[\dfrac{h(-2x-h)}{(x+h)^2x^2}\right]=\dfrac{-2x-h}{(x+h)^2x^2}$

R.7 #76. $\dfrac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}} = \dfrac{\dfrac{x+1-x}{x+1}}{\dfrac{2x-(x-1)}{x}} = \dfrac{\dfrac{1}{x+1}}{\dfrac{x+1}{x}} = \dfrac{1}{x+1}\cdot\dfrac{x}{x+1} = \dfrac{x}{(x+1)^2}$

R.7 D. $\dfrac{4+\dfrac{1}{x^2}}{25-\dfrac{1}{x^2}} = \dfrac{\dfrac{4x^2}{x^2}+\dfrac{1}{x^2}}{\dfrac{25x^2}{x^2}-\dfrac{1}{x^2}} = \dfrac{\dfrac{4x^2+1}{x^2}}{\dfrac{25x^2-1}{x^2}} = \dfrac{4x^2+1}{x^2}\cdot\dfrac{x^2}{25x^2-1} = \dfrac{x^2(4x^2+1)}{x^2(25x^2-1)} =$
$\dfrac{4x^2+1}{25x^2-1} = \dfrac{4x^2+1}{(5x+1)(5x-1)}$

R.8 A. Simplify: $2\sqrt{3}-\sqrt{48} = 2\sqrt{3}-\sqrt{16\cdot 3} = 2\sqrt{3}-\sqrt{16}\cdot\sqrt{3} = 2\sqrt{3}-4\cdot\sqrt{3} = -2\sqrt{3}$

R.8 B. $2\sqrt{3}+2\sqrt{12} = 2\sqrt{3}+2\sqrt{4\cdot 3} = 2\sqrt{3}+2\cdot 2\sqrt{3} = 2\sqrt{3}+4\sqrt{3} = 6\sqrt{3}$

R.8 #18. $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}} = \sqrt[3]{\dfrac{1}{27x^3}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{27}\sqrt[3]{x^3}} = \dfrac{1}{3x}$

R.8 #30. $2\sqrt{12}-3\sqrt{27} = 2\sqrt{4\cdot 3}-3\sqrt{9\cdot 3} = 2\cdot 2\sqrt{3}-3\cdot 3\sqrt{3} = 4\sqrt{3}-9\sqrt{3} = -5\sqrt{3}$

R.8 C. $4\sqrt[3]{7}-3\sqrt[3]{56} = 4\sqrt[3]{7}-3\sqrt[3]{8\cdot 7} = 4\sqrt[3]{7}-3\cdot 2\sqrt[3]{7} = 4\sqrt[3]{7}-6\sqrt[3]{7} = -2\sqrt[3]{7}$

R.8 D. $\dfrac{10}{4-\sqrt{2}} = \dfrac{10(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})} = \dfrac{10(4+\sqrt{2})}{16-2} = \dfrac{2\cdot 5\cdot(4+\sqrt{2})}{2\cdot 7} = \dfrac{5(4+\sqrt{2})}{7}$

R.8 E. $\left(\dfrac{1}{64}\right)^{-2/3} = \left(\dfrac{64}{1}\right)^{2/3} = 64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$

R.8 F. $\left(\dfrac{27}{8}\right)^{-2/3} = \left(\dfrac{8}{27}\right)^{2/3} = \dfrac{8^{2/3}}{27^{2/3}} = \dfrac{(\sqrt[3]{8})^2}{(\sqrt[3]{27})^2} = \dfrac{2^2}{3^2} = \dfrac{4}{9}$

R.8 G. $x^{1/2}(x^2+x)+x^{3/2}-24x^{1/2} = x^{1/2}(x^2+x)+x^{1/2}\cdot x-24x^{1/2} = x^{1/2}(x^2+x+x-24) =$
$x^{1/2}(x^2+2x-24) = x^{1/2}(x+6)(x-4)$

R.8 H. $(2\sqrt{x}-3)(2\sqrt{x}+5) = (2\sqrt{x})^2-6\sqrt{x}+10\sqrt{x}-15 = 4x+4\sqrt{x}-15$