MATH 110 REVIEW

to accompany

Sullivan: ***College Algebra****, 8th Ed.*

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Contents

Preface

This set of review materials contains a brief summary of each section, together with sample test questions that have been taken from previous exams actually given in Math 110. You can find the solutions to odd numbered problems from the text in the Student Solutions Manual.

John Beachy
Spring 2006

The review material has been updated for the Eighth Edition of **College Algebra**. Note that there are now solutions to the problems that are not in the Student Solutions Manual.

Fall 2007

Chapter 0

Review

Section summaries

Section R.1: Real Numbers

You should review the rules for working with numbers, especially those for fractions (see page 13):

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \qquad \text{where } b, c, d \text{ are nonzero}$$

Review problems: p16 #71,73,77,81,83

Section R.2: Algebra Essentials

In this section the rules for exponents are particularly important.

$$a^m a^n = a^{m+n} \qquad a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad \left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

Explanation: The shorthand notation that we use in writing a^3 instead of aaa dates back only to the 1600's, so it is a fairly recent invention (in the long history of math). When we multiply powers in this form, we can expand the powers and just count how many terms we have in the result. For example, $a^4 \cdot a^3 = aaaa \cdot aaa = a^7$, This method works for all positive exponents and leads to the general rule $a^m \cdot a^n = a^{m+n}$.

It's very useful to allow the exponents to be 0 or a negative number. But then there's no way to interpret the shorthand notation as a repeated product of a. We should at least make sure that any definition we give is consistent with the general formula $a^m \cdot a^n = a^{m+n}$.

To define a^0, we note that by the general rule for exponents we should have $a^m \cdot a^0 = a^{m+0} = a^m$. Dividing both sides of the equation by a^m gives us $\dfrac{a^m \cdot a^0}{a^m} = \dfrac{a^m}{a^m}$, and so we should have $a^0 = 1$. This shows that to be consistent with the rule for exponents we must define $a^0 = 1$.

If n is a positive integer, then to define a^{-n} we can give a similar argument. We should have $a^n \cdot a^{-n} = a^{n-n} = a^0 = 1$. Dividing both sides of the equation by a^n shows that we must define $a^{-n} = \dfrac{1}{a^n}$.

Review problems: p27 #71,87,91

Section R.3: Geometry Essentials

You need to know the **Pythagorean theorem**: $a^2 + b^2 = c^2$ for any right triangle with legs a and b and hypotenuse c.

Area of a rectangle = length · width;

area of a triangle = $\dfrac{1}{2}$ base · height;

area of a circle = $\pi \cdot (\text{radius})^2$.

Volume of a rectangular box = length · width · height.

Circumference of a circle = π · diameter.

Review problems: p36 #21,27,31,47,49

Section R.4: Polynomials

A **polynomial** is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$. If a_n is nonzero, then it is the **leading coefficient** and the polynomial has **degree** n. Polynomials are added by adding like terms, and multiplied by using the distributive laws. (A special case is to use FOIL to multiply $(ax + b)(cx + d)$.)

You need to remember that $(x+a)^2 = x^2+2ax+a^2$, and $(x+a)^3 = x^3+3ax^2+3a^2x+a^3$.

See Example 14 on page 46 for a good example of long division of polynomials.

Review problems: p48 #79,83,87,93,99,103

Section R.5: Factoring Polynomials

In this section there are a few formulas to remember. The factorization of the difference of two squares is given by $x^2 - a^2 = (x-a)(x+a)$. We also have $x^3 - a^3 = (x-a)(x^2+ax+a^2)$ and $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$, but $x^2 + a^2$ cannot be factored (using real numbers). The polynomials $x^2 + 2ax + a^2 = (x + a)^2$ and $x^2 - 2ax + a^2 = (x - a)^2$ are perfect squares.

Review problems: p56 #77,87,93,105,121

Section R.7: Rational Expressions

The quotient of two polynomials is called a **rational expression**. The rules for working with them are the same as for working with fractions (rational numbers).

$$\frac{a(x)}{b(x)} \cdot \frac{c(x)}{d(x)} = \frac{a(x)c(x)}{b(x)d(x)} \qquad \frac{a(x)}{b(x)} \div \frac{c(x)}{d(x)} = \frac{a(x)}{b(x)} \cdot \frac{d(x)}{c(x)} = \frac{a(x)d(x)}{b(x)c(x)}$$

$$\frac{a(x)}{b(x)} + \frac{c(x)}{d(x)} = \frac{a(x)d(x) + b(x)c(x)}{b(x)d(x)}$$

When you add or subtract rational expressions, it often turns out that using the product of their denominators makes the fraction more complicated than necessary. The least common multiple method (see page 65) can save a lot of work.

Review problems: p70 #45,71,81,85

Section R.8: Roots; Rational Exponents

The number b is the nth root of a, written $b = \sqrt[n]{a}$, if $b^n = a$. This can also be written as $b = a^{1/n}$. See page 73 for some properties of radicals.

Rational exponents (also called fractional exponents) can be evaluated using roots and powers:

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m \quad \text{or} \quad a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}.$$

Review problems: p78 #47,53,71,81,89,91

Sample Questions

R.2 #92. After simplifying, the **numerator** of $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y}$ is

(a) $2x^2$
(b) $2x^2z$
(c) yz
(d) 1
(e) 4

R.3 #15. The lengths of the legs of a right triangle are 7 and 24. Find the length of the hypotenuse.

(a) 84 (b) 31 (c) 25 (d) $\sqrt{97}$ (e) $\sqrt{62}$

R.3 A. Find the **volume** V and the **surface area** S of a rectangular box with length 4 feet, width 2 feet, and height 5 feet.

(a) $V = 40$ cubic feet and $S = 76$ square feet
(b) $V = 40$ cubic feet and $S = 11$ square feet
(c) $V = 20$ cubic feet and $S = 76$ square feet
(d) $V = 20$ cubic feet and $S = 11$ square feet
(e) None of these

R.3 #31. Find the **volume** V and **surface area** S of a rectangular box with length 8 feet, width 4 feet, and height 7 feet.

(a) $V = 214$ cubic feet and $S = 116$ square feet
(b) $V = 214$ cubic feet and $S = 232$ square feet
(c) $V = 224$ cubic feet and $S = 116$ square feet
(d) $V = 224$ cubic feet and $S = 232$ square feet
(e) None of these

R.4 A. Perform the indicated operation and simplify: $(2x - 5)(3x + 4)$

(a) $6x^2 - 20$
(b) $6x^2 - 7x - 20$
(c) $5x^2 - 7x - 20$
(d) $5x^2 - 2x - 20$
(e) $5x - 1$

R.4 B. Perform the indicated operation and simplify: $(2x^2)^3(4x^3)$

(a) $32x^9$
(b) $32x^8$
(c) $8x^8$
(d) $6x^8$
(e) None of these

R.4 C. Simplify: $(x^2 - 3x + 1) - (2x - 5)$

(a) $-2x^3 + 11x^2 - 17x + 5$

(b) $x^2 - x - 4$

(c) $x^2 - 5x - 4$

(d) $x^2 - 5x + 6$

(e) $x^2 - x + 6$

R.4 Example 9b. Expand and simplify: $(x - 1)^3$

(a) $x^3 - 1$

(b) $x^3 - x^2 - x + 1$

(c) $x^3 - 3x^2 - x + 1$

(d) $x^3 - 3x^2 + 3x - 1$

(e) None of these

R.4 #64. Multiply and simplify: $(x - 3y)(-2x + y)$

(a) $-2x^2 - 5xy - 3y^2$

(b) $-2x^2 - 5xy + 3y^2$

(c) $-2x^2 - 7xy - 3y^2$

(d) $-2x^2 - 7xy + 3y^2$

(e) None of these

R.4 #84. Expand and simplify: $(2x + 3y)^2$

(a) $2x^2 + 3y^2$

(b) $4x^2 + 9y^2$

(c) $4x^2 + 12xy + 9y^2$

(d) $4x^2 + 6xy + 9y^2$

(e) None of these

R.4 #87. Expand and simplify: $(2x + 1)^3$

(a) $8(x^3 + 1)$

(b) $8x^3 + 1$

(c) $8x^3 + 4x^2 + 2x + 1$

(d) $8x^3 + 12x^2 + 6x + 1$

(e) None of these

R.4 D. What is the **remainder** when $x^2 + x + 1$ is divided by $x - 2$?

(a) 7

(b) 6

(c) 5

(d) 4

(e) None of these

R.4 #93. When $5x^4 - 3x^2 + x + 1$ is divided by $x^2 + 2$ the **remainder** is

(a) $x + 27$

(b) $x + 25$

(c) $x + 15$

(d) $x + 13$

(e) None of these

R.4 #97. When $2x^4 - 3x^3 + x + 1$ is divided by $2x^2 + x + 1$ the **quotient** is

(a) $x^2 - 2x$

(b) $x^2 + 2x$

(c) $x^2 - 2x + \frac{1}{2}$

(d) $x^2 + 2x - \frac{1}{2}$

(e) None of these

R.4 #103. When $x^3 - a^3$ is divided by $x - a$ the **quotient** is

(a) $x^2 + ax + a^2$

(b) $x^2 - ax + a^2$

(c) $x^2 + a^2$

(d) $x^2 - a^2$

(e) None of these

R.5 Example 11. Factor the following expression completely: $x^2 - x - 12$

(a) $(x - 6)(x + 2)$

(b) $(x + 4)(x - 3)$

(c) $(x + 6)(x - 2)$

(d) $(x - 4)(x + 3)$

(e) None of these

R.5 A. Which one of the following is a factor of $4x^2 + 5x - 6$?

(a) $4x + 3$

(b) $x - 2$

(c) $4x - 3$

(d) $2x - 3$

(e) $4x + 5$

R.5 B. Factor completely: $ac^2 + 5bc^2 - a - 5b$

(a) $(a + 5b)(c + 1)(c - 1)$

(b) $(a - 5b)(c + 1)(c - 1)$

(c) $(a + 5b)(c + 1)^2$

(d) $(a - 5b)(c + 1)^2$

(e) None of these

R.5 #111. Factor completely: $3(x^2+10x+25)-4(x+5)$

(a) $3x^2+26x+95$

(b) $(x+5)(3x-19)$

(c) $(x+5)(3x+11)$

(d) $-(x+5)^2$

(e) None of these

R.5 #122. Factor completely: $4(x+5)^3(x-1)^2+2(x+5)^4(x-1)$

(a) $8(x+5)^7(x-1)^3$

(b) $2(x+5)^3(x-1)$

(c) $6(x+5)^3(x-1)(x+1)$

(d) $2(x+5)^3(x-1)(3x+7)$

(e) None of these

R.7 A. Reduce the following rational expression to lowest terms: $\dfrac{x^2+x-6}{x^2-4}$

(a) $\dfrac{x+3}{x+2}$

(b) $\dfrac{x+3}{x-2}$

(c) $\dfrac{x-6}{-4}$

(d) $x-2$

(e) None of these

R.7 B. Simplify; leave your answer in factored form: $\dfrac{1}{x-1}-\dfrac{2}{x+2}$

(a) $\dfrac{-1}{2x+1}$

(b) $\dfrac{2x+1}{(x+1)(x+2)}$

(c) -1

(d) $\dfrac{-x+4}{(x-1)(x+2)}$

(e) None of these

R.7 #45. After simplifying, the **numerator** of $\dfrac{4}{x-1}-\dfrac{2}{x+2}$ is

(a) 2

(b) -8

(c) $2(x+3)$

(d) $2(x-4)$

(e) None of these

R.7 #47. After simplifying, the **numerator** of $\dfrac{x}{x+1}+\dfrac{2x-3}{x-1}$ is

(a) $3(x-1)$

(b) $3x^2-2x+3$

(c) $3x^2-4x+3$

(d) $3x^2+4x+3$

(e) None of these

R.7 Example 10. Simplify (assuming $x \neq -3, 0$): $\dfrac{\dfrac{1}{2}+\dfrac{3}{x}}{\dfrac{x+3}{4}}$

(a) $\dfrac{2(x+2)}{x^2}$

(b) $\dfrac{8}{x(x+3)}$

(c) $\dfrac{(x+6)(x+3)}{8x}$

(d) $\dfrac{2(x+6)}{x(x+3)}$

(e) None of these

R.7 C. Simplify: $\dfrac{\left(\dfrac{x}{x+1}\right)}{\left(\dfrac{2x+2}{x^2}\right)}$

(a) $\dfrac{x^2}{2(x+2)}$

(b) $\dfrac{x^3}{2(x+1)^2}$

(c) $\dfrac{x^3}{2(x^2+1)}$

(d) $\dfrac{2}{x}$

(e) None of these

R.7 #66. After simplifying $\dfrac{2}{(x+2)^2(x-1)}-\dfrac{6}{(x+2)(x-1)^2}$, the **numerator** is

(a) $-2(4x+7)$

(b) $-2(4x-5)$

(c) $-2(2x-5)$

(d) $-2(2x+7)$

(e) $-2(2x-7)$

R.7 #72. After simplifying, the **numerator** of $\frac{1}{h}\left[\frac{1}{(x+h)^2}-\frac{1}{x^2}\right]$ is

(a) $-2x-h$

(b) $-2x+h$

(c) $2x-h$

(d) $2x+h$

(e) None of these

R.7 #76. After simplifying, the **denominator** of $\dfrac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}}$ is

(a) $x+1$

(b) $(x+1)^2$

(c) x^2-1

(d) $(x-1)^2$

(e) None of these

R.7 D. Simplify and factor: $\dfrac{4+\frac{1}{x^2}}{25-\frac{1}{x^2}}$.

(a) $\dfrac{4}{25}$

(b) $\dfrac{4x^2+1}{(x+5)(x-5)}$

(c) $\dfrac{(2x+1)(2x-1)}{(5x+1)(5x-1)}$

(d) $\dfrac{(x+2)(x-2)}{(x+5)(x-5)}$

(e) $\dfrac{4x^2+1}{(5x+1)(5x-1)}$

R.8 A. Simplify: $2\sqrt{3}-\sqrt{48}$

(a) $2\sqrt{3}$

(b) $-14\sqrt{3}$

(c) $-2\sqrt{3}$

(d) $3\sqrt{5}$

(e) -6

R.8 B. Simplify: $2\sqrt{3}+2\sqrt{12}$

(a) $2\sqrt{15}$

(b) $6\sqrt{3}$

(c) $10\sqrt{3}$

(d) 30

(e) None of these

R.8 Example 8a. Simplify: $(x^{2/3}y)(x^{-2}y)^{1/2}$

(a) $x^{8/3}$

(b) $\dfrac{y}{x^{2/3}}$

(c) $\dfrac{y^{3/2}}{x^{4/3}}$

(d) $\dfrac{y^{3/2}}{x^{1/3}}$

(e) None of these

R.8 Example 8c. Simplify: $\left(\dfrac{9x^2y^{1/3}}{x^{1/3}y}\right)^{1/2}$

(a) $3x$

(b) $\dfrac{3x^{9/5}}{y^{1/3}}$

(c) $\dfrac{3x^{5/6}}{y^{1/3}}$

(d) $\dfrac{9x^{9/5}}{y^{1/3}}$

(e) None of these

R.8 #18. Simplify (assuming that all variables are positive: $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}}$

(a) $\dfrac{1}{\sqrt{3x}}$

(b) $\dfrac{\sqrt{3}}{x}$

(c) $\dfrac{x}{\sqrt{3}}$

(d) $\dfrac{1}{3\sqrt{x}}$

(e) None of these

R.8 #30. Simplify: $2\sqrt{12} - 3\sqrt{27}$

(a) $-\sqrt{15}$

(b) $-19\sqrt{3}$

(c) $-5\sqrt{3}$

(d) $-6\sqrt{324}$

(e) None of these

R.8 C. Simplify: $4\sqrt[3]{7} - 3\sqrt[3]{56}$

(a) 2

(b) $-3\sqrt[3]{49}$

(c) $-4\sqrt[3]{7}$

(d) $2\sqrt[3]{7}$

(e) None of these

R.8 D. Rationalize the denominator: $\dfrac{10}{4-\sqrt{2}}$

(a) $5\sqrt{2}$

(b) $\dfrac{5(4-\sqrt{2})}{7}$

(c) $\dfrac{5(4+\sqrt{2})}{7}$

(d) $\dfrac{5(4-\sqrt{2})}{9}$

(e) $\dfrac{5(4+\sqrt{2})}{9}$

R.8 E. Simplify $\left(\dfrac{1}{64}\right)^{-2/3}$

(a) 16

(b) 512

(c) $-\frac{1}{16}$

(d) $-\frac{1}{96}$

(e) None of these

R.8 F. Simplify $\left(\dfrac{27}{8}\right)^{-2/3}$

(a) $\frac{9}{4}$

(b) $\frac{4}{9}$

(c) $\frac{-4}{9}$

(d) $\frac{-16}{81}$

(e) None of these

R.8 G. Factor the expression $x^{1/2}(x^2+x)+x^{3/2}-24x^{1/2}$ (where $x \geq 0$).

(a) $x^{1/2}(x+1)(x-3)$

(b) $x^{3/2}(x+1)(x-3)$

(c) $x^{3/2}(x+2)(x-6)$

(d) $x^{1/2}(x+6)(x-4)$

(e) $(x+6)(x-4)$

R.8 H. Multiply and simplify $(2\sqrt{x}-3)(2\sqrt{x}+5)$

(a) $4x+4\sqrt{x}-15$

(b) $2x+4\sqrt{x}-15$

(c) $4x-15$

(d) $4\sqrt{x}-15$

(e) None of these

R.8 #37. Simplify: $\sqrt[3]{16x^4} - \sqrt[3]{2x}$

(a) $\sqrt[3]{14}x$

(b) $(8x^3 - 1)\sqrt[3]{2x}$

(c) $(8x - 1)\sqrt[3]{2x}$

(d) $(2x - 1)\sqrt[3]{2x}$

(e) None of these

R.8 #53. After rationalizing the denominator of $\dfrac{\sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$, the numerator is

(a) $2x + h$

(b) $2x + h - 2\sqrt{x(x+h)}$

(c) $2x + h + 2\sqrt{x(x+h)}$

(d) $2x - 2\sqrt{x(x+h)}$

(e) None of these

Answer Key

R.2 #92. (d)

R.3 #15. (c)

R.3 A. (a)

R.3 #31. (d)

R.4 A. (b)

R.4 B. (a)

R.4 C. (d)

R.4 Example 9b. (d)

R.4 #64. (e)

R.4 #84. (c)

R.4 #87. (d)

R.4 D. (a)

R.4 #93. (a)

R.4 #97. (c)

R.4 #103. (a)

R.5 Example 11. (d)

R.5 A. (c)

R.5 B. (a)

R.5 #111. (c)

R.5 #122. (c)

R.7 A. (a)

R.7 B. (d)

R.7 #45. (e)

R.7 #47. (e)

R.7 Example 10. (d)

R.7 C. (b)

R.7 #66. (d)

R.7 #72. (a)

R.7 #76. (b)

R.7 D. (e)

R.8 A. (c)

R.8 B. (b)

R.8 Example 8a. (d)

R.8 Example 8c. (c)

R.8 #18. (e)

R.8 #30. (c)

R.8 C. (e)

R.8 D. (c)

R.8 E. (a)

R.8 F. (b)

R.8 G. (d)

R.8 H. (a)

R.8 #37. (d)

R.8 #53. (b)

Solutions

R.2 #92. $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y} = \dfrac{4}{8x^2x^4y(yz)} = \dfrac{1}{2x^6y^2z}$

R.3 A. Find the **volume** V and the **surface area** S of a rectangular box with length 4 feet, width 2 feet, and height 5 feet.

Solution: The volume is length times width times height, so we get $4 \cdot 2 \cdot 5 = 40$ cubic feet. To find the surface area: there are 4 sides, two that are 4×5 and two that are 2×5. The total area for the sides is $20 + 20 + 10 + 10 = 60$ square feet. The top and bottom of the box are 4×2, adding $8 + 8 = 16$ more square feet. The total surface area is 76 square feet. The answer is $V = 40$ cubic feet and $S = 80$ square feet.

R.4 A. $(2x-5)(3x+4) = (2x)(3x+4) - (5)(3x+4) = 6x^2 + 8x - 15x - 20 = 6x^2 - 7x - 20$

R.4 B. $(2x^2)^3(4x^3) = 2^3(x^2)^3(4x^3) = 8x^6 \cdot 4x^3 = (8 \cdot 4)(x^6x^3) = 32x^9$

R.4 C. $(x^2-3x+1)-(2x-5) = x^2-3x+1-2x+5 = x^2+(-3x-2x)+(1+5) = x^2-5x+6$

R.4 #64. $(x-3y)(-2x+y) = x(-2x+y) + (-3y)(-2x+y) = -2x^2 + xy + 6xy - 3y^2 = -2x^2 + 7xy - 3y^2$

R.4 #84. $(2x+3y)^2 = (2x+3y)(2x+3y) = 4x^2 + 6xy + 6xy + 9y^2 = 4x^2 + 12xy + 9y^2$

R.4 D. What is the **remainder** when x^2+x+1 is divided by $x-2$?

Solution: The remainder is +7.

$$\begin{array}{r|rrr} & & x & +3 \\ \cline{2-4} x \quad -2 & x^2 & +x & +1 \\ & x^2 & -2x & \\ \cline{2-3} & & +3x & +1 \\ & & +3x & -6 \\ \cline{3-4} & & & +7 \end{array}$$

R.5 A. $4x^2+5x-6=(4x-3)(x+2)$

R.5 B. $ac^2+5bc^2-a-5b=c^2(a+5b)-a-5b=c^2(a+5b)+(-1)(a+5b)=(c^2-1)(a+5b)=(c+1)(c-1)(a+5b)$

R.5 #122. $4(x+5)^3(x-1)^2+2(x+5)^4(x-1)=$

$(2\cdot 2)(x+5)^3(x-1)(x-1)+2(x+5)^3(x+5)(x-1)=$ (find common terms)

$2(x+5)^3(x-1)\cdot(2(x-1)+(x+5))=$ (factor out common terms)

$2(x+5)^3(x-1)(2x-2+x+5)=$

$2(x+5)^3(x-1)(3x+3)=$

$6(x+5)^3(x-1)(x+1)$

R.7 A. $\dfrac{x^2+x-6}{x^2-4}=\dfrac{(x+3)(x-2)}{(x+2)(x-2)}=\dfrac{x+3}{x+2}$

R.7 B. $\dfrac{1}{x-1}-\dfrac{2}{x+2}=\dfrac{x+2}{(x-1)(x+2)}-\dfrac{2(x-1)}{(x-1)(x+2)}=\dfrac{x+2-2x+2}{(x-1)(x+2)}=\dfrac{-x+4}{(x-1)(x+2)}$

R.7 C. $\dfrac{\left(\dfrac{x}{x+1}\right)}{\left(\dfrac{2x+2}{x^2}\right)}=\left(\dfrac{x}{x+1}\right)\left(\dfrac{x^2}{2(x+1)}\right)=\dfrac{x^3}{2(x+1)^2}$

R.7 #66. $\dfrac{2}{(x+2)^2(x-1)}-\dfrac{6}{(x+2)(x-1)^2}=\dfrac{2(x-1)}{(x+2)^2(x-1)^2}-\dfrac{6(x+2)}{(x+2)^2(x-1)^2}=$

$\dfrac{2x-2-6(x+2)}{(x+2)^2(x-1)^2}=\dfrac{2x-2-6x-12}{(x+2)^2(x-1)^2}=\dfrac{-4x-14}{(x+2)^2(x-1)^2}=\dfrac{-2(2x+7)}{(x+2)^2(x-1)^2}$

R.7 #72. $\dfrac{1}{h}\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]=\dfrac{1}{h}\left[\dfrac{x^2-(x+h)^2}{(x+h)^2x^2}\right]=\dfrac{1}{h}\left[\dfrac{x^2-(x^2+2xh+h^2)}{(x+h)^2x^2}\right]=$

$\dfrac{1}{h}\left[\dfrac{x^2-x^2-2xh-h^2}{(x+h)^2x^2}\right]=\dfrac{1}{h}\left[\dfrac{-2xh-h^2}{(x+h)^2x^2}\right]=\dfrac{1}{h}\left[\dfrac{h(-2x-h)}{(x+h)^2x^2}\right]=\dfrac{-2x-h}{(x+h)^2x^2}$

R.7 #76. $\dfrac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}} = \dfrac{\dfrac{x+1-x}{x+1}}{\dfrac{2x-(x-1)}{x}} = \dfrac{\dfrac{1}{x+1}}{\dfrac{x+1}{x}} = \dfrac{1}{x+1}\cdot\dfrac{x}{x+1} = \dfrac{x}{(x+1)^2}$

R.7 D. $\dfrac{4+\dfrac{1}{x^2}}{25-\dfrac{1}{x^2}} = \dfrac{\dfrac{4x^2}{x^2}+\dfrac{1}{x^2}}{\dfrac{25x^2}{x^2}-\dfrac{1}{x^2}} = \dfrac{\dfrac{4x^2+1}{x^2}}{\dfrac{25x^2-1}{x^2}} = \dfrac{4x^2+1}{x^2}\cdot\dfrac{x^2}{25x^2-1} = \dfrac{x^2(4x^2+1)}{x^2(25x^2-1)} =$
$\dfrac{4x^2+1}{25x^2-1} = \dfrac{4x^2+1}{(5x+1)(5x-1)}$

R.8 A. Simplify: $2\sqrt{3}-\sqrt{48} = 2\sqrt{3}-\sqrt{16\cdot 3} = 2\sqrt{3}-\sqrt{16}\cdot\sqrt{3} = 2\sqrt{3}-4\cdot\sqrt{3} = -2\sqrt{3}$

R.8 B. $2\sqrt{3}+2\sqrt{12} = 2\sqrt{3}+2\sqrt{4\cdot 3} = 2\sqrt{3}+2\cdot 2\sqrt{3} = 2\sqrt{3}+4\sqrt{3} = 6\sqrt{3}$

R.8 #18. $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}} = \sqrt[3]{\dfrac{1}{27x^3}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{27}\sqrt[3]{x^3}} = \dfrac{1}{3x}$

R.8 #30. $2\sqrt{12}-3\sqrt{27} = 2\sqrt{4\cdot 3}-3\sqrt{9\cdot 3} = 2\cdot 2\sqrt{3}-3\cdot 3\sqrt{3} = 4\sqrt{3}-9\sqrt{3} = -5\sqrt{3}$

R.8 C. $4\sqrt[3]{7}-3\sqrt[3]{56} = 4\sqrt[3]{7}-3\sqrt[3]{8\cdot 7} = 4\sqrt[3]{7}-3\cdot 2\sqrt[3]{7} = 4\sqrt[3]{7}-6\sqrt[3]{7} = -2\sqrt[3]{7}$

R.8 D. $\dfrac{10}{4-\sqrt{2}} = \dfrac{10(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})} = \dfrac{10(4+\sqrt{2})}{16-2} = \dfrac{2\cdot 5\cdot(4+\sqrt{2})}{2\cdot 7} = \dfrac{5(4+\sqrt{2})}{7}$

R.8 E. $\left(\dfrac{1}{64}\right)^{-2/3} = \left(\dfrac{64}{1}\right)^{2/3} = 64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$

R.8 F. $\left(\dfrac{27}{8}\right)^{-2/3} = \left(\dfrac{8}{27}\right)^{2/3} = \dfrac{8^{2/3}}{27^{2/3}} = \dfrac{(\sqrt[3]{8})^2}{(\sqrt[3]{27})^2} = \dfrac{2^2}{3^2} = \dfrac{4}{9}$

R.8 G. $x^{1/2}(x^2+x)+x^{3/2}-24x^{1/2} = x^{1/2}(x^2+x)+x^{1/2}\cdot x-24x^{1/2} = x^{1/2}(x^2+x+x-24) =$
$x^{1/2}(x^2+2x-24) = x^{1/2}(x+6)(x-4)$

R.8 H. $(2\sqrt{x}-3)(2\sqrt{x}+5) = (2\sqrt{x})^2-6\sqrt{x}+10\sqrt{x}-15 = 4x+4\sqrt{x}-15$

Chapter 1

Equations and Inequalities

Section summaries

Section 1.1 Linear Equations

A **linear equation** has the form $ax + b = 0$, where $a \neq 0$. It is solved by shifting b to the other side of the equation and dividing by a, to get $x = -\dfrac{b}{a}$.

Review problems: p95 #39,55,61,89

Section 1.2 Quadratic Equations

A **quadratic equation** has the form $ax^2 + bx + c = 0$, where $a \neq 0$. To solve a quadratic equation, first try to factor it and set each factor equal to zero. If you can't see factors right away, then use the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\,.$$

You *must* know the quadratic formula. If $b^2 - 4ac$ is negative, then we have the square root of a negative number, so in this case the equation has no solution (in the set of real numbers).

The quadratic formula is derived using the technique of **completing the square**. This technique will also be used in later sections, so you need to know how to use it. The technique is described on page 100; it is based on the fact that if you are given $x^2 + mx$, then adding $\left(\dfrac{m}{2}\right)^2$ gives $x^2 + mx + \left(\dfrac{m}{2}\right)^2$, which is equal to $\left(x + \dfrac{m}{2}\right)^2$.

Review problems: p107 #43,67,89,105,113

Section 1.4 Radical Equations

Radical equations are equations involving a square root, cube root, etc. Solving them involves raising each side of the equation to some power. Example 3 on page 119 provides a good illustration. In solving $\sqrt{2x+3} - \sqrt{x+2} = 2$ it might be tempting to square both sides, but the work is easier if you first solve for $\sqrt{2x+3}$, as shown in the example. Sometimes an equation may be a"disguised" quadratic equation. Making a substitution can be the easiest way to see the it looks like a quadratic. Then you solve it like you did in Section 1.2. The last step is to change back to the original variable. See Example 6 on page 121 for a good example of the technique.

Review problems: p122 #15,29,43,57,75

Section 1.5 Solving Inequalities

Read page 128 carefully. In solving an inequality, only a few things are different from solving an equation. If you multiply or divide both sides by a *negative* number, you must remember to reverse the direction of the inequality.

Review problems: p132 #59,77,83,91,107

Section 1.6 Equations and Inequalities Involving Absolute Value

The absolute value notation $| \quad |$ was introduced to find the distance between two points on the number line (review Example 3 on page 19).

An inequality like $|x| < a$ can be written as $|x - 0| < a$, and can be interpreted as saying that the distance between x and 0 is less than a. The solution on the number line gives the interval $-a < x < a$.

On the other hand, $a < |x|$ says that the distance between x and 0 is greater than a. On the number line, this translates into two intervals on either side of 0. Any value of x with $a < x$ will satisfy the inequality, but so will any value of x with $x < -a$.

Summary: $|u| < a$ can be simplified to the double inequality $-a < u < a$ while $|u| > a$ can be simplified to two inequalities: $u > a$ OR $u < -a$. (See page 136 and page 137.)

Review problems: p138 #41,45,47,63,73

Section 1.7 Problem Solving

For problems involving motion, use distance = rate · time. In some other types of problems, the principle is the same: rate · time gives you the total amount produced. This works for problems involving jobs.

It's hard to give general methods for such a variety of problems; use your common sense, and practice a lot of them.

Review problems: p146 #19,25,31,35,37,49

Sample Questions

1.1 A. Solve for x: $7 - 2x = 9 + 3x$

(a) $x = 2$
(b) $x = -2$
(c) $x = \frac{2}{5}$
(d) $x = -\frac{2}{5}$
(e) $x = -3$

1.1 B. Solve the equation: $1 - \frac{1}{2}x = 6 + x$.

(a) $x = \frac{10}{3}$
(b) $x = -\frac{10}{3}$
(c) $x = 2$
(d) $x = -3$
(e) None of these

1.1 Example 6. Solve the equation: $\dfrac{3x}{x-1} + 2 = \dfrac{3}{x-1}$

(a) $x = 1$
(b) $x = 5$
(c) $x = \frac{1}{5}$
(d) There is no solution
(e) None of these

1.1 #51. Solve this equation: $\dfrac{2x}{x^2-4} = \dfrac{4}{x^2-4} - \dfrac{3}{x+2}$

(a) $x = 2$ or $x = -2$
(b) $x = -1$
(c) $x = 2$
(d) There is no solution
(e) None of these

1.1 #59. Solve this equation: $\dfrac{4}{x-2} = \dfrac{-3}{x+5} + \dfrac{7}{x^2+3x-10}$

(a) $x = 2$
(b) $x = 1$
(c) $x = -\frac{19}{17}$
(d) There is no solution
(e) None of these

1.1 C. Going into the final exam, which will count as two tests, Brooke has test scores of 80, 83, 71, 61, and 89. What score does Brooke need on the final in order to have an average score of 80?

(a) 90
(b) 88
(c) 85
(d) 82
(e) None of these

1.1 #96. A wool suit, discounted by 30% for a clearance sale, has a price tag of $399. What was the suit's original price?

(a) Not enough information to determine
(b) $306.92 (approximately)
(c) $518.70
(d) $532
(e) $570

1.2 A.Solve for x: $x^2 - 3x + 2 = 0$

(a) $x = 1$ or $x = 2$
(b) $x = 1$ or $x = -2$
(c) $x = 2$ or $x = 3$
(d) $x = 2$ or $x = -3$
(e) None of these

1.2 B. Solve for x: $x^2 - 2x = 4$

(a) $x = 4$ or $x = 2$

(b) $x = \dfrac{4+\sqrt{5}}{2}$ or $x = \dfrac{2-\sqrt{5}}{2}$

(c) $x = \dfrac{2+\sqrt{5}}{2}$ or $x = \dfrac{4-\sqrt{5}}{2}$

(d) $x = 1 + \sqrt{5}$ or $x = 1 - \sqrt{5}$

(e) None of these

1.2 C. Find the value of a so that $x^2 + ax + \frac{1}{9}$ is a perfect square.

(a) $a = \frac{1}{3}$ (b) $a = \frac{2}{3}$ (c) $a = \frac{1}{9}$ (d) $a = \frac{2}{9}$ (e) None of these

1.2 D. Find the value of k so that $x^2 - \frac{3}{2}x + k$ is a perfect square.

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{9}{16}$ (d) $-\frac{9}{16}$ (e) None of these

1.2 #51. Use the quadratic formula to solve this equation: $\frac{2}{3}x^2 - \frac{5}{3}x + 1 = 0$

(a) $x = -1$ and $x = -2$

(b) $x = -1$ and $x = -\frac{3}{2}$

(c) $x = 1$ and $x = -\frac{3}{2}$

(d) $x = 1$ and $x = 2$

(e) None of these

1.2 #67. Find the real solutions, if any: $4 - \dfrac{1}{x} - \dfrac{2}{x^2} = 0$

(a) $x = \dfrac{-1 \pm \sqrt{17}}{8}$

(b) $x = \dfrac{1 \pm \sqrt{33}}{8}$

(c) $x = \dfrac{1}{2}$ or $x = -\dfrac{1}{4}$

(d) There is no solution

(e) None of these

1.2 #87. Use the quadratic formula to find the real solutions: $x^2 + \sqrt{2}x = \frac{1}{2}$

(a) $x = \dfrac{\sqrt{2} \pm 2}{2}$

(b) $x = \dfrac{-\sqrt{2} \pm 2}{2}$

(c) $x = \dfrac{\sqrt{2} \pm \sqrt{3}}{2}$

(d) $x = \dfrac{-\sqrt{2} \pm \sqrt{3}}{2}$

(e) None of these

1.2 #105. An open box is to be constructed from a square piece of sheet metal by removing a square of side 1 foot from each corner and turning up the edges. If the box is to hold 4 cubic feet, then the dimensions of the sheet metal should be

(a) 1 foot by 1 foot

(b) 2 feet by 2 feet

(c) 4 feet by 4 feet

(d) 8 feet by 8 feet

(e) None of these

1.2 #107. A ball is thrown vertically upward from the top of a building 96 feet tall with an initial velocity of 80 feet per second. The height s (in feet) of the ball above the ground after t seconds is $s = 96 + 80t - 16t^2$. After how many seconds does the ball strike the ground? That is, after how many seconds is the height equal to zero?

(a) $t = 1$
(b) $t = 2$
(c) $t = 3$
(d) $t = 6$
(e) None of these

1.2 E. The equation $1 - \frac{1}{x} - \frac{12}{x^2} = 0$ has

(a) NO real solutions
(b) exactly ONE real solution, which is POSITIVE
(c) exactly ONE real solution, which is NEGATIVE
(d) exactly TWO real solutions, whose product is -12
(e) exactly TWO real solutions, whose product is $-\frac{1}{12}$

1.4 A. Find the real solutions of the equation $\sqrt{x^2 + 16} = 5$

(a) There are exactly two real roots whose sum is 5
(b) There are exactly two real roots whose product is 5
(c) There are exactly two real roots whose sum is 0
(d) There is exactly one real root, which is positive
(e) There is exactly one real root, which is negative

1.4 #25. Find the real solutions: $3 + \sqrt{3x + 1} = x$

(a) $x = 1$ or $x = -8$
(b) $x = 1$ or $x = 8$
(c) $x = -8$
(d) $x = 8$
(e) None of these

1.4 #28. The solution to the equation $\sqrt{3x + 7} + \sqrt{x + 2} = 1$ is

(a) $x = -1$
(b) $x = -2$
(c) $x = -1$ or $x = -2$
(d) There is no solution
(e) None of these

1.4 #31. The solution to the equation $\sqrt{3-2\sqrt{x}} = \sqrt{x}$ is

(a) $x = 9$
(b) $x = 1$ or $x = 9$
(c) $x = 3$ or $x = -3$
(d) There is no solution
(e) None of these

1.5 A. The solution to the inequality $9x - 5 < 6x + 1$ is

(a) $x < 2$
(b) $x > 2$
(c) $x < \frac{2}{5}$
(d) $x > \frac{2}{5}$
(e) None of these

1.5 #75. The solution set of the inequality $1 < 1 - \frac{1}{2}x < 4$ is the interval

(a) $(-6, 0)$
(b) $(0, 6)$
(c) $[0, 6]$
(d) $[-6, 0]$
(e) None of these

1.5 #80. The solution set of the inequality $x(9x-5) \leq (3x-1)^2$ is

(a) $[1, \infty)$
(b) $(-\infty, 1]$
(c) $[\frac{1}{5}, \infty)$
(d) $\{0, \frac{5}{9}, \frac{1}{3}\}$
(e) The empty set

1.5 #87. The solution set of the inequality $0 < (2x-4)^{-1} < \frac{1}{2}$ is the interval

(a) $(3, \infty)$
(b) $(-\infty, 3)$
(c) $(0, 3)$
(d) $(0, \infty)$
(e) None of these

1.5 B. The solution set of the inequality $0 < (x-4)^{-1} < \frac{1}{2}$ is the interval

(a) $(0, 2)$
(b) $(0, 6)$
(c) $(6, \infty)$
(d) $(-\infty, 6)$
(e) $(2, 3)$

1.6 A. The solution set of the combined inequality $-1 < 3 - 2x \leq 15$ is

(a) $(-6, 2]$
(b) $[-6, 2)$
(c) $(2, 6]$
(d) $[2, 6)$
(e) $[-13/2, \infty)$

1.6 B. Solve this inequality: $|x - 2| < 3$

(a) $-2 < x < 1$
(b) $-2 < x < 5$
(c) $2 < x < 5$
(d) $0 < x < 1$
(e) None of these

1.6 Example 6. Solve this inequality: $|2x - 5| > 3$

(a) $x < 1$ or $x > 4$
(b) $x < -1$ or $x > 4$
(c) $x < 4$ and $x > 1$
(d) $x < 4$ and $x > -1$
(e) None of these

1.6 C. Find the solution set of this inequality: $|5 - 2x| < 9$

(a) The empty set (no solutions)
(b) $\{x \mid x > 7 \text{ or } x < -2\}$
(c) $(-2, 7)$
(d) $(-2, \infty)$
(e) $(-\infty, 7)$

1.6 D. Find the solution set of this inequality: $|3 - 2x| \geq 7$

(a) $\{x \mid x \leq 5 \text{ or } x \geq -2\}$
(b) $\{\ x \mid\ x \geq 5 \text{ or } x \leq -2\}$
(c) $[-2, 5]$
(d) all real numbers
(e) $(-\infty, -2]$

1.6 #43. The solution set of the inequality $\mid 2x - 3 \mid \geq 2$ is

(a) $\{x \mid \frac{1}{2} \leq x \leq \frac{5}{2}\}$
(b) $\{x \mid x \leq -\frac{1}{2} \text{ or } x \geq \frac{5}{2}\}$
(c) $\{x \mid x \leq \frac{1}{2} \text{ or } x \geq \frac{5}{2}\}$
(d) $\{x \mid -\frac{1}{2} \leq x \leq \frac{5}{2}\}$
(e) None of these

1.6 #45. Solve this inequality: $|1 - 4x| - 7 < -2$

(a) $-\frac{3}{2} < x < 1$
(b) $-\frac{3}{2} < x < -1$
(c) $1 < x < \frac{3}{2}$
(d) $-1 < x < \frac{3}{2}$
(e) None of these

1.7 #27. A motorboat maintains a constant speed of 15 miles per hour relative to the water in going 10 miles upstream and then returning. If the total time for the trip is 1.5 hours, then the speed of the current must be

(a) 2 miles per hour

(b) 4 miles per hour

(c) 5 miles per hour

(d) 10 miles per hour

(e) None of these

1.7 A. (see #28) Two cars enter the Florida Turnpike at Commercial Boulevard at 8:00 AM, each heading for Wildwood. One car's average speed is 5 miles per hour more than the other's. The slower car arrives at Wildwood at 11:00 AM, 15 minutes after the other car. What is the average speed of the slower car?

(a) 50 mph

(b) 55 mph

(c) 60 mph

(d) 70 mph

(e) None of these

1.7 #33. Trent can deliver his newspapers in 30 minutes. It takes Lois 20 minutes to do the same route. How long would it take them to deliver the newspapers if they work together?

(a) 50 minutes

(b) 25 minutes

(c) 12 minutes

(d) 10 minutes

(e) None of these

1.7 B. If x gallons of cherry juice costing \$3.00 per gallon are to be combined with y gallons of apple juice costing \$1.00 per gallon to make a fruit juice mix costing \$2.50 per gallon, then what is $\frac{x}{y}$?

(a) 3

(b) 10

(c) $\frac{1}{3}$

(d) $\frac{8}{5}$

(e) It cannot be determined

Answer Key

1.1 A. (d)

1.1 B. (b)

1.1 Example 6. (d)

1.1 #51. (d)

1.1 #59. (e)

1.1 C. (b)

1.1 #96. (e)

1.2 A.(a)

1.2 B. (d)

1.2 C. (b)

1.2 D. (c)

1.2 #51. (e)

1.2 #67. (b)

1.2 #87. (b)

1.2 #105. (c)

1.2 #107. (d)

1.2 E. (d)

1.4 A. (c)

1.4 #25. (d)

1.4 #28. (b)

1.4 #31. (e)

1.5 A. (a)

1.5 #75. (a)

1.5 #80. (b)

1.5 #87. (a)

1.5 B. (c)

1.6 A. (b)

1.6 B. (e)

1.6 Example 6. (a)

1.6 C. (c)

1.6 D. (b)

1.6 #43. (c)

1.6 #45. (d)

1.7 #27. (c)

1.7 A. (b)

1.7 #33. (c)

1.7 B. (a)

Solutions

1.1 A. Solve for x: $7 - 2x = 9 + 3x$

Solution: $7 - 2x = 9 + 3x$ $\quad 7 - 9 = 3x + 2x$ $\quad 5x = -2$ $\quad x = -\frac{2}{5}$

1.1 B. Solve the equation: $1 - \frac{1}{2}x = 6 + x.$

Solution: $1 - \frac{1}{2}x = 6 + x.$ $\quad 2 - x = 12 + 2x.$ $\quad -10 = 3x.$ $\quad x = -\frac{10}{3}.$

1.1 C. Going into the final exam, which will count as two tests, Brooke has test scores of 80, 83, 71, 61, and 89. What score does Brooke need on the final in order to have an average score of 80?

Solution: If x is the final exam score, her average will be $(80+83+71+61+89+2x)/7$, since the final counts double. She needs $(80 + 83 + 71 + 61 + 89 + 2x)/7 = 80$, so she must have $80+83+71+61+89+2x = 560$, which reduces to $384+2x = 560$, or $2x = 176$. The answer is $x = 88$.

1.1 #96. A wool suit, discounted by 30% for a clearance sale, has a price tag of \$399. What was the suit's original price?

Solution: Since the suit has been discounted by 30%, the sale price is 70% of the original price. If we let x be the original price, then we get the equation $0.7x = 399$, and dividing both sides by 0.7 gives the answer: \$570.

1.2 A.Solve for x: $x^2 - 3x + 2 = 0$

Solution: This can be solved by factoring. $x^2 - 3x + 2 = 0$ $\quad (x - 2)(x - 1) = 0$
The only way a product of real numbers can be zero is if one of the numbers is zero, so either $x - 2 = 0$ or $x - 1 = 0$ and this gives us the answer: $x = 1$ or $x = 2$.

1.2 B. Solve for x: $x^2 - 2x = 4$

Solution: Put all terms on the left hand side of the equation and try to factor.
It is hard to see a factorization for $x^2 - 2x - 4 = 0$ so the next choice is to use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 1$, $b = -2$, and $c = -4$. This gives us

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot (-4)}}{2} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm \sqrt{4 \cdot 5}}{2} = \frac{2 \pm \sqrt{4}\sqrt{5}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}.$$

1.2 C. Find the value of a so that $x^2 + ax + \frac{1}{9}$ is a perfect square.

Solution: The general theory is that if you square $x - c$, you get $x^2 + 2cx + c^2$. If you know the last coefficient, then to get the middle one you take the square root and multiply by 2. In this problem, take the square root of $\frac{1}{9}$, which gives you $\frac{1}{3}$, and double it. The answer: $a = \frac{2}{3}$. Check: $(x - \frac{1}{3})^2 = (x - \frac{1}{3})(x - \frac{1}{3}) = x^2 + \frac{2}{3}x + \frac{1}{9}$

1.2 D. Find the value of k so that $x^2 - \frac{3}{2}x + k$ is a perfect square.

Solution: This time (see the previous solution) we know the middle coefficient and need to find the last one. Remember the form $x^2 + 2cx + c^2$. We need to divide the middle coefficient by 2, and then square to get the last coefficient. Divide $\frac{3}{2}$ by 2 (multiplying by $\frac{1}{2}$ gives the same answer) to get $\frac{3}{2}$, and then squaring gives us the answer: $\frac{9}{16}$.
Check: $(x - \frac{3}{4})^2 = (x - \frac{3}{4})(x - \frac{3}{4}) = x^2 - \frac{6}{4}x + \frac{9}{16}$

1.2 E. The equation $1 - \dfrac{1}{x} - \dfrac{12}{x^2} = 0$ has

Solution: We cannot have $x = 0$, so we can multiply through by x^2, to get $x^2 - x - 12 = 0$. This equation can be solved by factoring, since $x^2 - x - 12 = (x-4)(x+3)$, so we get $x = 4$ or $x = -3$. Answer: there are exactly TWO real solutions, whose product is -12.

1.4 A. Find the real solutions of the equation $\sqrt{x^2 + 16} = 5$

Solution: $\sqrt{x^2 + 16} = 5$ $\quad x^2 + 16 = 25$ $\quad x^2 = 9$ $\quad x = \pm 3$

Check (since the equation was squared): $\sqrt{(\pm 3)^2 + 16} = \sqrt{9 + 16} = \sqrt{25} = 5$

1.4 #28. The solution to the equation $\sqrt{3x+7} + \sqrt{x+2} = 1$ is

Solution: First move $\sqrt{x+2}$ to the other side of the equation, then square both sides. At the end, you must check the answers carefully because squaring the equation may have introduced an extra solution.

$\sqrt{3x+7} + \sqrt{x+2} = 1 \qquad \sqrt{3x+7} = 1 - \sqrt{x+2} \qquad (\sqrt{3x+7})^2 = (1 - \sqrt{x+2})^2$

$3x + 7 = (1 - \sqrt{x+2})(1 - \sqrt{x+2}) = 1 - 2\sqrt{x+2} + (\sqrt{x+2})^2 = 1 - 2\sqrt{x+2} + x + 2$

$3x + 7 = x + 3 - 2\sqrt{x+2} \qquad 2x + 4 = 2\sqrt{x+2} \qquad x + 2 = \sqrt{x+2}$

Now we need to square both sides again. $(x+2)^2 = (\sqrt{x+2})^2 \qquad x^2 + 4x + 4 = x + 2$

$x^2 + 3x + 2 = 0 \qquad (x+2)(x+1) = 0 \qquad x + 2 = 0$ or $x + 1 = 0 \qquad x = -2$ or $x = -1$

Check these answers: $\sqrt{3(-2) + 7} + \sqrt{(-2) + 2} = \sqrt{1} + \sqrt{0} = 1$, so -2 is a solution.

$\sqrt{3(-1) + 7} + \sqrt{(-1) + 2} = \sqrt{4} + \sqrt{1} = 3$, so -1 is *not* a solution. Final answer: $x = -2$.

1.5 A. The solution to the inequality $9x - 5 < 6x + 1$ is $\{x \mid x < 2\}$.

Solution: $9x - 5 < 6x + 1 \quad 9x - 6x < 1 + 5 \quad 3x < 6 \quad x < 2$

1.5 #80. The solution set of the inequality $x(9x - 5) \le (3x - 1)^2$ is $(-\infty, 1]$.

Solution: $x(9x - 5) \le (3x - 1)^2 \qquad 9x^2 - 5x \le 9x^2 - 6x + 1 \qquad x \le 1$ $\qquad$ Answer: $(-\infty, 1]$

1.5 B. The solution set of the inequality $0 < (x-4)^{-1} < \frac{1}{2}$ is

Solution: First note that we must have $x - 4 > 0$. Then we can invert the inequality $\dfrac{1}{x-4} < \dfrac{1}{2}$ to get $x - 4 > 2$. (Remember that inverting the terms reverses the inequality. For example, $\frac{1}{4} < \frac{1}{2}$, but $4 > 2$.) The final answer is $(6, \infty)$.

1.6 A. The solution set of the combined inequality $-1 < 3 - 2x \le 15$ is

Solution: $-1 < 3 - 2x \le 15$ $\quad -4 < -2x \le 12$ $\quad 4 > 2x \ge -12$ $\quad 2 > x \ge -6$
Now $-6 \le x < 2$, so the final answer (in interval form) is $[-6, 2)$.

1.6 B. Solve this inequality: $|x-2| < 3$

Solution: $|x-2| < 3$ $\quad -3 < x - 2 < 3$ $\quad -1 < x < 5$

1.6 C. Find the solution set of this inequality: $|5 - 2x| < 9$

Solution: $|5 - 2x| < 9$ $\quad -9 < 5 - 2x < 9$ $\quad -14 < -2x < 4$ $\quad -4 < 2x < 14$
$-2 < x < 7$ Answer (in interval form): $(-2, 7)$

1.6 D. Find the solution set of this inequality: $|3 - 2x| \ge 7$

Solution: We need to replace $|3-2x| \ge 7$ with two inequalities: $3-2x \ge 7$ or $3-2x \le -7$.
$-2x \ge 4$ or $-2x \le -10$ $\quad x \le -2$ or $x \ge 5$ $\quad$ In set notation: $\{\ x \mid\ x \ge 5 \text{ or } x \le -2\}$

1.7 A. (see #28) Two cars enter the Florida Turnpike at Commercial Boulevard at 8:00 AM, each heading for Wildwood. One car's average speed is 5 miles per hour more than the other's. The slower car arrives at Wildwood at 11:00 AM, 15 minutes after the other car. What is the average speed of the slower car?

Solution: Let x be the average speed of the slower car. Use the fact that both cars travel the same distance and the formula $rate \cdot time = distance$, with distance in miles and time in hours. Distance for the slower car: $3x$. Distance for the faster car: $(2\frac{3}{4})(x+5)$.
We get this equation: $3x = (2\frac{3}{4})(x+5)$.
Solve: $3x = (2\frac{3}{4})x + (\frac{11}{4})(5)$ $\quad (3 - 2\frac{3}{4})x = \frac{55}{4}$ $\quad \frac{1}{4}x = \frac{55}{4}$ $\quad x = 55$

1.7 B. If x gallons of cherry juice costing \$3.00 per gallon are to be combined with y gallons of apple juice costing \$1.00 per gallon to make a fruit juice mix costing \$2.50 per gallon, then what is $\dfrac{x}{y}$?

Solution: The equation that describes the total cost of the mixture is $3x+y = 2.5(x+y)$.
Divide both sides by y to find the ratio: $\dfrac{3x}{y} + \dfrac{y}{y} = \dfrac{2.5x}{y} + \dfrac{2.5y}{y}$.
Simplify: $(3)\dfrac{x}{y} + 1 = (2.5)\dfrac{x}{y} + 2.5$ $\quad (.5)\dfrac{x}{y} = 1.5$ $\quad \dfrac{x}{y} = 3$

Chapter 2

Graphs

Section summaries

Section 2.1 The Distance and Midpoint Formulas

You need to know the **distance formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the **midpoint formula**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) .$$

The distance formula comes from the Pythagorean theorem (review page 30); you may also need to use the Pythagorean theorem to verify that three points are the vertices of a right triangle.

Review problems: p161 #19,29,35,45

Section 2.2 Graphs of Equations

Review the procedure for finding x and y-intercepts on page 166. Review the tests for symmetry on page 168. A function is **even** precisely when its graph is symmetric with respect to the y-axis; it is **odd** precisely when its graph is symmetric with respect to the origin. (Compare the tests on page 168 to the tests on pages 231 and 232.)

Review problems: p171 #41,43,63,65,67

Section 2.3 Lines

The **slope** of the line segment joining two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} ,$$

assuming that $x_1 \neq x_2$. The equation of the line through (x_1, y_1) with slope m is

$$y = m(x - x_1) + y_1 ,$$

the **point-slope form**. The equation of the line with slope m and y-intercept b is

$$y = mx + b ,$$

the **slope-intercept form**. To find the slope of a line in **general form** $Ax + By = C$, put it into the slope-intercept form so you can just read off the slope. Remember that a positive slope means that the graph goes up (from left to right) and a negative slope means that the graph heads down.

Two different lines $y = m_1x + b_1$ and $y = m_2x + b_2$ are **parallel** when $m_2 = m_1$, and **perpendicular** when $m_2 = -\frac{1}{m_1}$ (or, equivalently, when $m_1m_2 = -1$).

Review problems: p185 #21,29,57,85,87,113,115,119,131

Section 2.4 Circles

The **standard form** of an equation of a circle with radius r and center (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2 .$$

If you are given an equation in the **general form** $x^2 + y^2 + ax + by + c = 0$, you can complete the square to put it into the standard form.

Review problems: p193 #15,19,29,59

Sample Questions

2.1 A. Find the distance between the points $(2, 5)$ and $(4, -3)$.

(a) $2\sqrt{2}$
(b) 10
(c) $\sqrt{10}$
(d) $2\sqrt{17}$
(e) 68

2.1 B. Find the distance between the points $(-1, -3)$ and $(2, 1)$.

(a) 1
(b) $\sqrt{5}$
(c) $\sqrt{17}$
(d) 25
(e) None of these

2.1 C. The midpoint of the line segment joining the points $(1, 6)$ and $(-3, 4)$ is

(a) $(\frac{1}{2}, \frac{7}{2})$
(b) $(\frac{7}{2}, \frac{1}{2})$
(c) $(-1, 5)$
(d) $(-2, -1)$
(e) $(16, 4)$

2.1 #48. Find all points on the y-axis that are 5 units from the point $(4, 4)$.

(a) $(-1, 0)$ and $(-7, 0)$
(b) $(0, 5)$ and $(0, 5)$
(c) $(0, 1)$ and $(0, 7)$
(d) $(0, -1)$ and $(0, -7)$
(e) None of these

2.2 #25. The graph of the line with equation $2x + 3y = 6$ has

(a) x-intercept $(3, 0)$ and y-intercept $(0, 2)$
(b) x-intercept $(2, 0)$ and y-intercept $(0, 3)$
(c) x-intercept $(2, 0)$ and y-intercept $(0, 6)$
(d) x-intercept $(6, 0)$ and y-intercept $(0, 3)$
(e) x-intercept $(6, 0)$ and y-intercept $(0, 2)$

2.2 #59. Find the x-intercepts of the graph of the equation $x^2 + y - 9 = 0$.

(a) The x-intercepts are $\sqrt{3}$ and $-\sqrt{3}$
(b) The only x-intercept is -9
(c) The x-intercepts are 3 and -3
(d) The only x-intercept is 3
(e) None of these

2.2 #61. The graph of the equation $9x^2 + 4y^2 = 36$ has

(a) x-intercept $(0, 0)$ and y-intercept $(0, 0)$

(b) x-intercept $(2, 0)$ and y-intercept $(0, 3)$

(c) x-intercept $(3, 0)$ and y-intercept $(0, 2)$

(d) x-intercepts $(2, 0)$ and $(-2, 0)$ and y-intercepts $(0, 3)$ and $(0, -3)$

(e) x-intercepts $(3, 0)$ and $(-3, 0)$ and y-intercepts $(0, 2)$ and $(0, -2)$

2.2 #69. The graph of $y = \dfrac{-x^3}{x^2 - 9}$ is symmetric with respect to

(a) the x-axis and y-axis, but NOT the origin.

(b) the origin, but NOT the x-axis or y-axis.

(c) the x-axis and the origin, but NOT the y-axis.

(d) the y-axis and origin, but NOT the x-axis.

(e) the x-axis, the y-axis and the origin.

2.3 A. The equation of the vertical line passing through the point $(4, 7)$ is

(a) $x = 4$

(b) $x = 7$

(c) $y = 4$

(d) $y = 7$

(e) $4x = 7y$

2.3 B. Find the slope of the line through the points $(-3, -1)$ and $(1, 7)$.

(a) 3

(b) -3

(c) $\frac{1}{2}$

(d) 2

(e) None of these

2.3 C. Find an equation for the line through $(0, 3)$ and $(-2, 0)$.

(a) $2x - 3y + 6 = 0$

(b) $3x + 2y - 6 = 0$

(c) $3x - 2y + 6 = 0$

(d) $2x + 3y - 6 = 0$

(e) $3x + 2y + 6 = 0$

2.3 Example 8. Find the slope m and y-intercept b of the equation $2x + 4y = 8$.

(a) $m = \frac{1}{2}$ and $b = 2$

(b) $m = -\frac{1}{2}$ and $b = 2$

(c) $m = 2$ and $b = 4$

(d) $m = -2$ and $b = 4$

(e) None of these

2.3 #49. The equation of the line containing the points $(1, 3)$ and $(-1, 2)$ is

(a) $y = 2x + 1$

(b) $y = -\frac{1}{2}x + \frac{7}{2}$

(c) $y = \frac{1}{2}x + \frac{5}{2}$

(d) $y = -2x + 5$

(e) This is a vertical line, so there is no equation.

2.3 D. Which of the following is an equation of the line passing through the point $(5, -4)$ and parallel to the line with equation $3x - 5y + 2 = 0$?

(a) $y = 3x - 4$

(b) $y = 3x - 19$

(c) $y = \frac{3}{5}x - 7$

(d) $y = \frac{3}{5}x - 4$

(e) $y = -\frac{5}{3}x - 9$

2.3 #65. Find an equation for the line perpendicular to $y = \dfrac{1}{2}x + 4$ containing $(1, -2)$.

(a) $y = 2x + 4$

(b) $y = -2x - 4$

(c) $y = 2x$

(d) $y = -2x$

(e) None of these

2.3 #67. Find an equation for the line perpendicular to $2x + y = 2$ and containing $(-3, 0)$.

(a) $y = 2(x + 3)$

(b) $y = -2(x + 3)$

(c) $y = \frac{1}{2}(x + 3)$

(d) $y = -\frac{1}{2}(x + 3)$

(e) None of these

2.3 E. The line which is perpendicular to the line given by $y = 4x - 3$ and which passes through the point $(0, 5)$ also passes through which of the following points?

(a) $(4, 0)$

(b) $(4, 13)$

(c) $(4, 4)$

(d) $(4, 6)$

(e) $(4, -11)$

2.3 #97. The graph of the line with equation $\frac{1}{2}x + \frac{1}{3}y = 1$ has

(a) x-intercept $(1/2, 0)$ and y-intercept $(0, 1/3)$

(b) x-intercept $(1/3, 0)$ and y-intercept $(0, 1/2)$

(c) x-intercept $(3, 0)$ and y-intercept $(0, 2)$

(d) x-intercept $(2, 0)$ and y-intercept $(0, 3)$

(e) None of these

2.4 A. The standard form of the equation of the circle with radius 6 and center $(-3,-6)$ is

(a) $(x+3)^2+(y+6)^2=36$

(b) $(x-3)^2+(y-6)^2=36$

(c) $(x+6)^2+(y+3)^2=36$

(d) $(x-6)^2+(y-3)^2=36$

(e) None of these

2.4 #25. The circle $x^2+y^2-2x+4y-4=0$ has

(a) center $(1,-2)$ and radius 9

(b) center $(1,-2)$ and radius 3

(c) center $(-2,4)$ and radius 16

(d) center $(-1,2)$ and radius 3

(e) center $(-1,2)$ and radius 9

2.4 #29. The graph of the equation $x^2+y^2-x+2y+1=0$ is

(a) a circle with center $(1,-2)$ and radius 1.

(b) a circle with center $(-1,2)$ and radius 1.

(c) a circle with center $(\frac{1}{2},-1)$ and radius 1.

(d) a circle with center $(\frac{1}{2},-1)$ and radius $\frac{1}{4}$.

(e) None of these

2.4 B. The graph of the equation $x^2+y^2-6x+2y+7=0$ is

(a) a circle with center $(3,-1)$ and radius 3.

(b) a circle with center $(3,-1)$ and radius $\sqrt{3}$.

(c) a circle with center $(3,1)$ and radius $\sqrt{7}$.

(d) a circle with center $(1,3)$ and radius $\sqrt{3}$.

(e) None of these

Answer Key

2.1 A. (d)

2.1 B. (e)

2.1 C. (c)

2.1 #48. (c)

2.2 #25. (a)

2.2 #59. (c)

2.2 #61. (d)

2.2 #69. (b)

2.3 A. (a)

2.3 B. (d)

2.3. C. (c)

2.3 Example 8. (b)

2.3 #49. (c)

2.3 D. (c)

2.3 #65. (d)

2.3 #67. (c)

2.3 E. (c)

2.3 #97. (d)

2.4 A. (a)

2.4 #25. (b)

2.4 #29. (e)

2.4 B. (b)

Solutions

2.1 A. Find the distance between the points $(2, 5)$ and $(4, -3)$.
Solution: Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
with $x_2 = 4$, $x_1 = 2$, $y_2 = -3$, $y_1 = 5$.
$d = \sqrt{(4-2)^2 + (-3-5)^2} = \sqrt{(2)^2 + (-8)^2} = \sqrt{4+64} = \sqrt{68} = \sqrt{2 \cdot 34} = \sqrt{2 \cdot 2 \cdot 17}$
Answer: $d = 2\sqrt{17}$

2.1 B. Find the distance between the points $(-1, -3)$ and $(2, 1)$.
Solution: $d = \sqrt{(2-(-1))^2 + (1-(-3))^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

2.1 C. The midpoint of the line segment joining the points $(1,6)$ and $(-3,4)$ is

Solution: Use the midpoint formula $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$, which just averages the x-coordinates and the y-coordinates. $\left(\frac{1-3}{2}, \frac{6+4}{2}\right) = \left(\frac{-2}{2}, \frac{10}{2}\right) = (-1,5)$

2.1 #48. Find all points on the y-axis that are 5 units from the point $(4,4)$.

Solution: For a point to be on the y-axis its x-coordinate must be zero. Let $(0,y)$ be the point we are looking for, and use the distance formula: $\sqrt{(0-4)^2+(y-4)^2} = 5$
Solve for y: $16+(y-4)^2 = 25$ $(y-4)^2 = 9$ $y-4 = \pm 3$ $y = 1$ or $y = 7$
The two possible points are $(0,1)$ and $(0,7)$.

2.3 A. The equation of the vertical line passing through the point $(4,7)$ is

Solution: The points on the line all have the same x-coordinate, so the equation is $x = 4$.

2.3 B. Find the slope of the line through the points $(-3,-1)$ and $(1,7)$.

Solution: $m = \dfrac{y_2-y_1}{x_2-x_1} = \frac{7-(-1)}{1-(-3)} = \frac{8}{4} = 2$

2.3 C. Find an equation for the line through $(0,3)$ and $(-2,0)$.

Solution: These are the choices: (a) $2x-3y+6=0$ (b) $3x+2y-6=0$
(c) $3x-2y+6=0$ (d) $2x+3y-6=0$ (e) $3x+2y+6=0$
You can do this problem by just substituting into the equations. The point $(0,3)$ lies on lines (b) and (c), since these are the only equations that satisfy $x=0$, $y=3$. Of these two, only (c) satisfies $x=-2$ and $y=0$, so the answer must be equation (c).

You can also solve the problem by using the point-slope form. The slope is $m = \frac{0-3}{-2-0} = \frac{3}{2}$, and the y-intercept is 3 since the line goes through $(0,3)$. This gives the equation $y = \frac{3}{2}x+3$. Multiply through by 2 to get $2y = 3x+6$, or $0 = 3x-2y+6$.

2.3 D. Which of the following is an equation of the line passing through the point $(5,-4)$ and parallel to the line with equation $3x-5y+2=0$?

Solution: To be parallel to the given line, the slope must be the same. Convert the given equation into point-slopt form: $3x+2 = 5y$ or $y = \frac{3}{5}x+\frac{2}{5}$. The slope is $\frac{3}{5}$.
The choices are (a) $y = 3x-4$ (b) $y = 3x-19$ (c) $y = \frac{3}{5}x-7$
(d) $y = \frac{3}{5}x-4$ (e) $y = -\frac{5}{3}x-9$
Only (c) and (d) have the correct slope. The point $(5,-4)$ lies on line (c).

Knowing the slope, you could also use the point-slope form of the equation of a line:
$y = \frac{3}{5}(x-5)+(-4)$ $y = \frac{3}{5}x-3-4$ $y = \frac{3}{5}x-7$

2.3 E. The line which is perpendicular to the line given by $y = 4x-3$ and which passes through the point $(0,5)$ also passes through which of the following points?

Solution: The slope of the perpendicular line must be $-\frac{1}{4}$, and its y-intercept is 5 since it passes through $(0,5)$, so its equation is $y = -\frac{1}{4}x+5$. If $x=4$, then $y=4$, so the answer is (c).

2.4 A. The standard form of the equation of the circle with radius 6 and center $(-3, -6)$ is

Solution: Use the standard form of an equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$.
You get $(x+3)^2 + (y+6)^2 = 36$.

2.4 B. The graph of the equation $x^2 + y^2 - 6x + 2y + 7 = 0$ is

Solution: The answer is found by completing the square.

$x^2 - 6x + \;??\; + y^2 + 2y + \;??\; = -7$

$x^2 - 6x + 9 + y^2 + 2y + 1 = -7 + 9 + 1$

$(x-3)^2 + (y+1)^2 = 3$

$(x-3)^2 + (y-(-1))^2 = (\sqrt{3})^2$ This is a circle with center $(3, -1)$ and radius $\sqrt{3}$.

Chapter 3

Functions and Their Graphs

Section summaries

Section 3.1 Functions

A function from a set X to a set Y is a rule or correspondence that associates with each element of X exactly one element of Y. (In Math 110, these sets usually consist of real numbers.) With the function notation $y = f(x)$, each x value has only one corresponding y value.

You can think of a function as being like a program. The x-values are the inputs, and the y-values are the outputs. The possible inputs form the domain of the function, and the possible outputs form its range. For the functions that we are dealing with, the numbers that we need to exclude from the domain are numbers that lead to division by zero, or the square root of a negative number. (If the function was a program, trying to divide by zero or take the square root of a negative number would give an error message.)

Given two functions, we can make a new function from their sum, difference, product, or quotient. (See pages 217–218.)

Review problems: p219 #39,41,51,59,61,75,79,89

Section 3.2 The Graph of a Function

The **vertical line test**: A set of points in the (x, y)-plane is the graph of a function precisely when every vertical line intersects the set in at most one point.

Review problems: p226 #9,25,27,29

Section 3.3 Properties of Functions

A function is called **even** if $f(-x) = f(x)$ (the graph is symmetric about the y-axis) and **odd** if $f(-x) = -f(x)$ (the graph that is symmetric about the origin).

A function is **increasing** on an interval if its values keep going up, and **decreasing** on an interval if its values keep going down. A high point on the graph is called a **local**

maximum, and this corresponds to a change from increasing to decreasing. A low point on the graph is called a **local minimum**, and corresponds to a change from decreasing to increasing. *Note: Since we are not using calculators, you won't be asked to actually compute local maximum and local minimum values in this section.*

The average rate of change of a function is found by dividing the change in y by the change in x. If you go from x to c on the x-axis, then the corresponding change in y is $f(x) - f(c)$. We get this formula for the average rate of change: $\dfrac{f(x) - f(c)}{x - c}$, where $x \neq c$.

Review problems: p 239 #33,35,39,43,53,61,63

Section 3.4 A Library of Functions

This section gives the graphs of some functions you need to be able to recognize and to graph on your own.

Straight lines: $f(x) = mx + b$

The square function: $f(x) = x^2$

The cube function: $f(x) = x^3$

The square root function: $f(x) = \sqrt{x}$

The cube root function: $f(x) = \sqrt[3]{x}$

The reciprocal function: $f(x) = \dfrac{1}{x}$

The absolute value function: $f(x) = |x|$

There is no reason that a function has to have the same formula at each point in its domain. Of course, which formula is used for which numbers has to be spelled out very carefully. These functions get their name from being defined in "pieces". Example: the absolute value function $f(x) = |x|$ takes any number and makes it non-negative. It is convenient to express this with two different formulas: if x is already positive or zero, we don't need to make any change. But if x is negative, we need to change the sign.

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Review problems: p249 #19,21,25,31,37

Section 3.5 Graphing Techniques: Transformations

The basic model for a linear function is $f(x) = x$, whose graph is a straight line through the origin that slopes up at a 45^o angle. The family of linear functions includes all functions of the form $f(x) = ax + b$. We can get all of these by multiplying the basic example by a and adding b. The numbers a and b tell us all about the new line: if a is positive it slopes up, if a is negative it slopes down; b gives the y-intercept, and tells how far the line has been moved up (if $b > 0$) or down (if $b < 0$).

In Section 3.4, besides linear functions, we studied the basic examples of several families of functions:

quadratic functions	$f(x) = x^2$	cubic functions	$f(x) = x^3$
square root functions	$f(x) = \sqrt{x}$	cube root functions	$f(x) = \sqrt[3]{x}$
reciprocal functions	$f(x) = \dfrac{1}{x}$	absolute value functions	$f(x) = \mid x \mid$

In Section 3.5 we study the graphs that we get when the basic examples in each family are shifted up or down, shifted left or right, stretched or compressed, or reflected about one of the axes. We start with a function $f(x)$, and positive numbers a, h, and k, where $a > 1$. Changing the function does the following:

$f(x) + k$	shift up by k	$f(x) - k$	shift down by k
$f(x - h)$	shift right by h	$f(x + h)$	shift left by h
$af(x)$	stretch vertically	$\dfrac{1}{a}f(x)$	compress vertically
$-f(x)$	reflect about the x-axis	$f(-x)$	reflect about the y-axis

Review problems: p261 #27,32,53,59,69

Section 3.6 Mathematical Models

Some of the models involve geometry. You should review the Pythagorean theorem on page 30, and the geometry formulas on page 31 (for the area of a rectangle, a triangle, or a circle). You need to know that the volume of a box is its length times width times height. On the exam, if you need any other formulas for volumes, the question will include the formula.

There are also some basic models related to economics. You need to remember that when items are sold the revenue (money taken in) is found by multiplying the number of items sold by the price per unit. That's just common sense, and you shouldn't have to make a big deal about remembering the formula $R(x) = px$, where x is the number sold and p is the price per unit.

Some of the problems in the text ask you to find the model and then find a maximum or minimum value, using a calculator. Obviously, though this is an important idea, we will not test you on it. But if it happens that the model gives you a quadratic function, then it *is* a fair question to ask you to find the maximum or minimum value on the exam, because you can use the techniques from Section 4.3.

Review problems: p280 #5,8,15,23

Sample Questions

3.1 A. For the function $f(x) = x^3 + x$, find $f(-2)$.

(a) 6
(b) 10
(c) -6
(d) -10
(e) None of these

3.1 Example 6. For the function $f(x) = 2x^2 - 3x$, find $f(3x)$.

(a) $36x^2 - 9x$
(b) $36x^2 - 3x$
(c) $18x^2 - 9x$
(d) $18x^2 - 3x$
(e) None of these

3.1 B. For the function $f(x) = x^2 - 2$, find $f(y+2)$.

(a) $x^2y + 2x^2 - 2y - 4$
(b) $y^2 + 4y + 2$
(c) y^2
(d) $y^2 + 2$
(e) None of these

3.1 #51. What is the domain of the function $f(x) = \dfrac{x}{x^2 - 16}$?

(a) All real numbers
(b) All real numbers except 0
(c) All real numbers except $4, -4$
(d) All real numbers except 16
(e) None of these

3.1 C. What is the domain of the function $f(x) = \dfrac{x+1}{x-1}$?

(a) All real numbers except -1
(b) All real numbers except $1, -1$
(c) All real numbers except 1
(d) All real numbers
(e) None of these.

3.1 D. What is the domain of the function $g(x) = \dfrac{x^2+1}{x^3 - 4x}$?

(a) All real numbers except -4
(b) All real numbers except $2, -2$
(c) All real numbers except $0, 2, -2$
(d) $\{0, 2, -2\}$
(e) $\{0, 2, -2, -4\}$

3.1 #57. Find the domain of the function $f(x) = \dfrac{4}{\sqrt{x-9}}$?

(a) $(9, \infty)$

(b) $[9, \infty)$

(c) $(4/9, \infty)$

(d) $(-\infty, 9)$

(e) $(-\infty, 9]$

3.1 E. What is the domain of the function $f(x) = \dfrac{x+2}{\sqrt{5+3x}}$?

(a) $(-\infty, 0]$

(b) $(-\infty, 0)$

(c) $(-2, \infty)$

(d) $(-5/3, \infty)$

(e) $(-\infty, -5/3)$

3.1 F. Let $f(x) = 2x^2 - 4$. Find $f(x-3)$.

(a) $2x^2 - 12x + 18$

(b) $2x^2 - 12x + 14$

(c) $2x^2 - 22$

(d) $2x^2 + 14$

(e) None of these

3.1 G. If $f(x) = \dfrac{2x+1}{3x-5}$, then what is $f(3x+1)$?

(a) $f(3x+1) = 3 \cdot \left(\dfrac{2x+1}{3x-5}\right) + 1$

(b) $f(3x+1) = \dfrac{6x+2}{9x-4}$

(c) $f(3x+1) = \dfrac{6x+3}{9x-2}$

(d) $f(3x+1) = \dfrac{(2x+1)(3x+1)}{(3x-5)}$

(e) None of these

3.1 #75. For $f(x) = x^2 - x + 4$, find and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$, where $h \neq 0$.

(a) $h + 1$

(b) $h - 1$

(c) $2x + h + 1$

(d) $2x + h - 1$

(e) None of these

3.1 #79. For $f(x) = x^3 - 2$, find and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$, where $h \neq 0$

(a) h^2

(b) $x^2 + xh + h^2$

(c) $3x^2 + 3xh + h^2$

(d) $3x^2 + 3xh^2 + h^3$

(e) None of these

3.2 #25c. For the function $f(x) = \dfrac{x+2}{x-6}$, if $f(x) = 2$, then $x =$

(a) -1

(b) 8

(c) 14

(d) There is no answer

(e) None of these

3.2 #25e. Find the x-intercept(s) of the graph of the function $f(x) = \dfrac{x+2}{x-6}$.

(a) 2

(b) -2

(c) $2, -6$

(d) $-1/3$

(e) None of these

3.2 #25f. Find the y-intercept(s) of the graph of the function $f(x) = \dfrac{x+2}{x-6}$.

(a) 2

(b) -2

(c) $2, -6$

(d) $-1/3$

(e) None of these

3.2 #27c. For the function $f(x) = \dfrac{2x^2}{x^4+1}$, if $f(x) = 1$, then $x =$

(a) 1

(b) -1

(c) $1, -1$

(d) There is no answer

(e) None of these

3.2 #27e. Find the x-intercept(s) of the graph of the function $f(x) = \dfrac{2x^2}{x^4+1}$.

(a) 0

(b) 1

(c) 2

(d) There is no x-intercept

(e) None of these

3.2 #27f. Find the y-intercept(s) of the graph of the function $f(x) = \dfrac{2x^2}{x^4+1}$.

(a) 0

(b) 1

(c) 2

(d) There is no y-intercept

(e) None of these

3.3 #39. The function $f(x) = x + |x|$ is

(a) Odd
(b) Even
(c) Neither odd nor even
(d) Both odd and even
(e) None of these

3.3 #41. The function $f(x) = \dfrac{1}{x^2}$ is

(a) Odd
(b) Even
(c) Neither odd nor even
(d) Both odd and even
(e) None of these

3.3 #43. The function $f(x) = \dfrac{-x^3}{3x^2 - 9}$ is

(a) Odd
(b) Even
(c) Neither odd nor even
(d) Both odd and even
(e) None of these

3.3 A. If f is an odd function and (a, b) lies on the graph of f, what other point(s) must also lie on the graph of f?

(a) $(-a, b)$
(b) $(-a, -b)$
(c) $(a, -b)$
(d) $(a, -b)$ and $(-a, b)$
(e) $(a, -b)$ and $(-a, -b)$

3.3 #55. For the function $f(x) = x^3 - 2x + 1$, find the average rate of change from $x = -1$ to $x = 1$.

(a) 0
(b) 1
(c) -1
(d) 2
(e) None of these

3.3 B. For the function $f(x) = \dfrac{4}{x^2}$, the equation of the secant line joining $(1, f(1))$ and $(2, f(2))$ is:

(a) $y = 3x + 1$
(b) $y = 3x - 5$
(c) $y = -3x + 7$
(d) $y = -3x + 13$
(e) None of these

3.3 C. For $f(x) = \sqrt{x}$, the equation of the line joining $(1, f(1))$ and $(4, f(4))$ is

(a) $y = \frac{1}{3}x + \frac{2}{3}$

(b) $y = \frac{1}{3}x + \frac{10}{3}$

(c) $y = 3x$

(d) $y = 3x - 2$

(e) None of these

3.4 A. Find the x-intercept(s) and the y-intercept of the function

$$f(x) = \begin{cases} 3 + x & \text{if } -3 \leq x < 0 \\ 2 & \text{if } x = 0 \\ \sqrt{x} - 1 & \text{if } 0 < x \end{cases}$$

(a) $x = -3$ and $y = 2$

(b) $x = -3$ and $y = 3$

(c) $x = 3$ and $y = 2$

(d) $x = 3$ and $y = 3$

(e) None of these

3.4 #25. If $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ 2x + 1 & \text{if } 0 < x \end{cases}$, find $f(x + 3)$ when $x > 0$.

(a) $x^2 + 6x + 9$

(b) $x^2 + 6x + 10$

(c) $2x + 6$

(d) $2x + 7$

(e) None of these

3.4 #35. If $f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \end{cases}$, find the range of $f(x)$.

(a) $(-\infty, 1)$

(b) $(0, \infty)$

(c) $[0, \infty)$

(d) $(-\infty, \infty)$

(e) None of these

3.5 A. Using the function $f(x) = x^3$, find the equation of the corresponding function whose graph is shifted 2 units to the right then shifted up 5 units.

(a) $f(x) = (x + 2)^3 + 5$

(b) $f(x) = (x - 2)^3 + 5$

(c) $f(x) = (x + 5)^3 + 2$

(d) $f(x) = (x - 5)^3 + 2$

(e) None of these

3.5 B. Using the function $f(x) = x^3$, find the equation of the corresponding function whose graph is shifted left 1 unit, reflected about the x-axis, and shifted up 2 units.

(a) $f(x) = -(x+2)^3 + 1$

(b) $f(x) = -(x-2)^3 + 1$

(c) $f(x) = -(x+1)^3 + 2$

(d) $f(x) = -(x-1)^3 + 2$

(e) None of these

3.5 C. Using the function $f(x) = x^3$, find the equation of the corresponding function whose graph is reflected about the x-axis and then shifted down 4 units.

(a) $f(x) = -(x-4)^3$

(b) $f(x) = -(x+4)^3$

(c) $f(x) = -x^3 + 4$

(d) $f(x) = -x^3 - 4$

(e) None of these

3.5 D. If the graph of $y = 3x^2 + 4x - 5$ is reflected about the y-axis, the new equation is

(a) $y = -3x^2 - 4x + 5$

(b) $y = -3x^2 + 4x + 5$

(c) $y = 3x^2 - 4x + 5$

(d) $y = 3x^2 - 4x - 5$

(e) None of these

3.5 E. Which function represents the graph at the right?

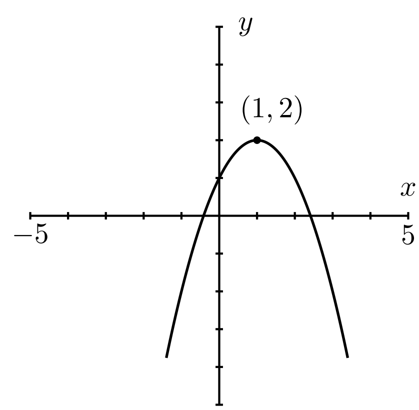
(a) $f(x) = -(x-1)^2 + 2$

(b) $f(x) = -(x-2)^2 + 1$

(c) $f(x) = -(x-1)^2 - 2$

(d) $f(x) = -(x-2)^2 - 1$

(e) None of these



3.6 #7. A rectangle has one corner on the graph of $y = 16 - x^2$, another at the origin, a third on the positive y-axis, and the fourth on the positive x-axis. Express the area as a function of x.

(a) $A(x) = -x^2 + x + 16$

(b) $A(x) = -x^3 + 16x$

(c) $A(x) = -x^3 + 8x$

(d) $A(x) = -x^2 + 16$

(e) None of these

3.6 #13. A wire of length x is bent into the shape of a circle. Express the area $A(x)$ as a function of x.

(a) $A(x) = x^2$

(b) $A(x) = \pi x^2$

(c) $A(x) = \dfrac{x^2}{4}$

(d) $A(x) = \dfrac{x^2}{4\pi}$

(e) None of these

3.6 A. Alex has 400 feet of fencing to enclose a rectangular garden. One side of the garden lies along the barn, so only three sides require fencing. Express the area $A(x)$ of the rectangle as a function of x, where x is the length of the side perpendicular to the side of the barn.

(a) $A(x) = -x^2 + 200x$

(b) $A(x) = -x^2 + 400x$

(c) $A(x) = -2x^2 + 200x$

(d) $A(x) = -2x^2 + 400x$

(e) None of these

3.6 B. Germaine has 40 feet of fencing to enclose a rectangular pool. One side of the pool lies along the house, so only three sides require fencing. Express the area $A(x)$ of the rectangle as a function of x, where x is the length of the side perpendicular to the side of the house.

(a) $A(x) = -x^2 + 40x$

(b) $A(x) = -x^2 + 20x$

(c) $A(x) = -2x^2 + 40x$

(d) $A(x) = -2x^2 + 20x$

(e) None of these

Answer Key

3.1 A. (d)

3.1 Example 6. (c)

3.1 B. (b)

3.1 #51. (c)

3.1 C. (c)

3.1 D. (c)

3.1 #57. (a)

3.1 E. (d)

3.1 F. (b)

3.1 G. (c)

3.1 #75. (d)

3.1 #79. (c)

3.2 #25c. (c)

3.2 #25e. (b)

3.2 #25f. (d)

3.2 #27c. (c)

3.2 #27e. (a)

3.2 #27f. (a)

3.3 #39. (c)

3.3 #41. (b)

3.3 #43. (a)

3.3 A. (b)

3.3 #55. (c)

3.3 B. (c)

3.3 C. (a)

3.4 A. (e)

3.4 #25. (d)

3.4 #35. (d)

3.5 A. (b)

3.5 B. (c)

3.5 C. (d)

3.5 D. (d)

3.5 E. (a)

3.6 #7. (b)

3.6 #13. (d)

3.6 A. (d)

3.6 B. (c)

Solutions

3.1 A. For the function $f(x) = x^3 + x$, find $f(-2)$.
Solution: (d) $f(-2) = (-2)^3 + (-2) = -8 - 2 = -10$

3.1 B. For the function $f(x) = x^2 - 2$, find $f(y+2)$.
Solution: (b) $f(y+2) = (y+2)^2 - 2 = (y+2)(y+2) - 2 = (y^2 + 4y + 4) - 2 = y^2 + 4y + 2$

3.1 C. What is the domain of the function $f(x) = \dfrac{x+1}{x-1}$?
Solution: (c) We only need to worry about division by 0, so we need to exclude $x = 1$ and the answer is: All real numbers except 1.

3.1 D. What is the domain of the function $g(x) = \dfrac{x^2+1}{x^3-4x}$?
Solution: (c) Set the denominator equal to 0 to see which numbers to exclude.
$x^3 - 4x = 0 \qquad x(x^2 - 4) = 0 \qquad x(x-2)(x+2) = 0$
Answer: All real numbers except $0, 2, -2$.

3.1 E. What is the domain of the function $f(x) = \dfrac{x+2}{\sqrt{5+3x}}$?
Solution: (d) We need to exclude numbers that lead to division by zero, or to the square root of a negative number. Solve: $0 < 5 + 3x \qquad -5 < 3x \qquad -\frac{5}{3} < x$
Answer (in interval notation): $(-5/3, \infty)$

3.1 F. Let $f(x) = 2x^2 - 4$. Find $f(x-3)$.
Solution: (b) $f(x-3) = 2(x-3)^2 - 4 = 2(x^2 - 6x + 9) - 4 = 2x^2 - 12x + 18 - 4$
Answer: $f(x-3) = 2x^2 - 12x + 14$

3.1 G. If $f(x) = \dfrac{2x+1}{3x-5}$, then what is $f(3x+1)$?
Solution: (c) You can think of the function this way: $f(\quad) = \dfrac{2(\quad)+1}{3(\quad)-5}$
Now substitute $3x+1$ into the empty slots that used to be x.
$$f(3x+1) = \frac{2(3x+1)+1}{3(3x+1)-5} = \frac{(6x+2)+1}{(9x+3)-5} = \frac{6x+3}{9x-2}$$

3.3 A. If f is an odd function and (a, b) lies on the graph of f, what other point(s) must also lie on the graph of f?

Solution: (b) The point symmetric about the origin must be on the graph, and this is the point $(-a, -b)$. Another way to see this is that if you substitute in the negative value $x = -a$, then the y value must change sign too.

3.3 B. For the function $f(x) = \dfrac{4}{x^2}$, the equation of the secant line joining $(1, f(1))$ and $(2, f(2))$ is:

Solution: (c) First find the slope m of the line segment joining the two points.
$f(1) = \dfrac{4}{1^2} = 4$ and $f(2) = \dfrac{4}{2^2} = 1$ so $m = \dfrac{f(2) - f(1)}{2-1} = \dfrac{1-4}{1} = -3$
This reduces the choices to (c) $y = -3x + 7$ or (d) $y = -3x + 13$. Substituting $x = 1$ into equation (c) does give the correct y value of 4, so it must be the right equation.

3.3 C. For $f(x) = \sqrt{x}$, the equation of the line joining $(1, f(1))$ and $(4, f(4))$ is

Solution: (a) $m = \dfrac{f(4) - f(1)}{4-1} = \dfrac{\sqrt{4} - \sqrt{1}}{4-1} = \dfrac{1}{3}$ so the answer could be (a) $y = \frac{1}{3}x + \frac{2}{3}$ or (b) $y = \frac{1}{3}x + \frac{10}{3}$. Equation (a) is correct since $f(1) = 1$.

3.4 A. Find the x-intercept(s) and the y-intercept of the function

$$f(x) = \begin{cases} 3 + x & \text{if } -3 \le x < 0 \\ 2 & \text{if } x = 0 \\ \sqrt{x} - 1 & \text{if } 0 < x \end{cases}$$

Solution: (e) To find the y-intercept, substitute $x = 0$ into the formula. This means we need to use the middle formula, so $y = 2$.
To find the x-intercepts, we need to solve the first and last equations for $y = 0$. In the first equation we get $3 + x = 0$, so $x = -3$. This is a legal value in the domain of this part of the formula, so it does give us an x-intercept. In the last equation, setting $\sqrt{x} - 1 = 0$ gives us $x = 1$, and again this is a legal value in the domain of this part of the formula.
Answer: $x = -3$, $x = 1$, and $y = 2$, which is not one of the choices (a) – (d).

3.5 A. Using the function $f(x) = x^3$, find the equation of the corresponding function whose graph is shifted 2 units to the right then shifted up 5 units.

Solution: (b) $f(x) = (x-2)^3 + 5$

3.5 B. Using the function $f(x) = x^3$, find the equation of the corresponding function whose graph is shifted left 1 unit, reflected about the x-axis, and shifted up 2 units.

Solution: (c) Shift left one unit: $f(x) = (x+1)^3$ Then reflect about the x-axis: $f(x) = -(x+1)^3$ Finally shift up 2 units: $f(x) = -(x+1)^3 + 2$

3.5 C. Using the function $f(x) = x^3$, find the equation of the corresponding function whose graph is reflected about the x-axis and then shifted down 4 units.

Solution: (d) $f(x) = -x^3 - 4$

3.5 D. If the graph of $y = 3x^2 + 4x - 5$ is reflected about the y-axis, the new equation is
Solution: (d) Substitute $-x$ to get $y = 3(-x)^2 + 4(-x) - 5 = 3x^2 - 4x - 5$

3.5 E. Which function represents the graph at the right?

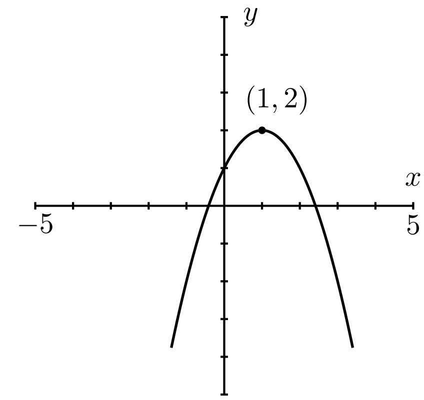
(a) $f(x) = -(x-1)^2 + 2$

(b) $f(x) = -(x-2)^2 + 1$

(c) $f(x) = -(x-1)^2 - 2$

(d) $f(x) = -(x-2)^2 - 1$

(e) None of these



Solution: (a) The graph is that of $y = x^2$ shifted 1 unit to the right, reflected about the x-axis, and then shifted up 2 units.

3.6 A. Alex has 400 feet of fencing to enclose a rectangular garden. One side of the garden lies along the barn, so only three sides require fencing. Express the area $A(x)$ of the rectangle as a function of x, where x is the length of the side perpendicular to the side of the barn.
Solution: (d) The three sides are x, x, and $400-2x$, so the total area is length $\times$ width, giving $A(x) = x(400-2x) = 400x - 2x^2 = -2x^2 + 400x$.

3.6 B. Germaine has 40 feet of fencing to enclose a rectangular pool. One side of the pool lies along the house, so only three sides require fencing. Express the area $A(x)$ of the rectangle as a function of x, where x is the length of the side perpendicular to the side of the house.
Solution: (c) The three sides are x, x, and $40 - 2x$, giving $A(x) = x(40-2x) = 40x - 2x^2 = -2x^2 + 40x$.

Chapter 4

Linear and Quadratic Functions

Section summaries

Section 4.1 Linear Functions and Their Properties

A **linear function** is one of the form

$$f(x) = mx + b\ ,$$

where m gives the slope of its graph, and b gives the y-intercept of its graph. The slope m measures the rate of growth of the function, so a linear function is increasing if $m > 0$ and decreasing if $m < 0$.

Review problems: p284 #17,21,25,37,43,49

Section 4.2 Building Linear Functions from Data

In this section linear functions are constructed from data presented in various ways.

Review problems: p290 #3,5,7,15,19,21

Section 4.3 Quadratic Functions and Their Properties

The **general form of a quadratic function** is

$$f(x) = a(x - h)^2 + k\ ,$$

where (h, k) is the vertex of the graph (which is a parabola). You can see from the formula that h gives the left/right shift while k gives the up/down shift. The coefficient a represents a vertical stretch or compression. Since the basic member of this family is $f(x) = x^2$, whose graph opens up, the graph of $f(x) = a(x - h)^2 + k$ will open up if a is positive, and down if a is negative. If the graph opens up, its height is minimum at the vertex; if the graph opens down, its height is maximum at the vertex.

If a quadratic is given in the form $f(x) = ax^2 + bx + c$, then the x-coordinate of its vertex is $x = -\frac{b}{2a}$. Since you already know the quadratic formula, you can remember it as part of the formula:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$$

This way of looking at the quadratic formula shows that if the graph has x-intercepts, they occur as points on either side of the line $x = -\frac{b}{2a}$, which is the **axis of symmetry** of the graph. The summary on page 301 explains the steps in graphing a quadratic function.

Review problems: p302 #13,15,27,31,37,43,55,61,81,83

Sample Questions

4.1 #29. Solve $f(x) \leq g(x)$ for $f(x) = 4x - 1$ and $g(x) = -2x + 5$.

(a) $(-\infty, 2)$
(b) $(-\infty, 2]$
(c) $(-\infty, 1)$
(d) $(-\infty, 1]$
(e) None of these

4.1 #37c. The cost (in dollars) of renting a truck is $C(x) = 0.25x+35$, where x is the number of miles driven. If you want the cost to be no more than $100, what is the maximum number of miles you can drive?

(a) 60
(b) 260
(c) 400
(d) 540
(e) None of these

4.3 A. If $f(x)$ is a quadratic function whose graph has the vertex (h, k), which one is the correct form of the function?

(a) $f(x) = a(x - h) + k$
(b) $f(x) = a(x - h)^2 + k$
(c) $f(x) = a(x - k)^2 + h$
(d) $f(x) = \sqrt{r^2 - (x - h)^2} + k$
(e) $f(x) = a(x - k)^3 + h$

4.3 #42 If $f(x) = x^2 - 2x - 3$, then the vertex of the graph of $f(x)$ is

(a) (-2,5)
(b) (-1,0)
(c) (1,-4)
(d) $(2, -3)$
(e) None of these

4.3 B. Find the vertex of the quadratic function $f(x) = 2x^2 - 4x + 9$.

(a) $(0, 9)$
(b) $(1, 1)$
(c) $(-1, 7)$
(d) $(1, 7)$
(e) None of these

4.3 C. Find the axis of symmetry of the graph of $f(x) = 4x^2 - 8x + 3$.

(a) $x = 1$
(b) $x = -1$
(c) $x = 2$
(d) $x = -2$
(e) None of these

4.3 D. Let $f(x) = 4x^2 - 8x + 3$. Find the x and y-intercepts, if any.

(a) $(-\frac{3}{2}, 0)$ $(-\frac{1}{2}, 0)$ $(0, 3)$
(b) $(\frac{3}{2}, 0)$ $(\frac{1}{2}, 0)$ $(0, 3)$
(c) $(-1, 0)$ $(0, 3)$
(d) $(15, 0)$ $(0, 3)$
(e) None of these

4.3 #55. Find the equation of the quadratic function whose graph has vertex $(-3, 5)$ and y-intercept -4.

(a) $f(x) = -(x - 3)^2 + 5$
(b) $f(x) = (x - 3)^2 - 13$
(c) $f(x) = -(x + 3)^2 + 5$
(d) $f(x) = (x + 3)^2 + 5$
(e) None of these

4.3 #61. Find the minimum value of the function $f(x) = 2x^2 + 12x - 3$.

(a) -57
(b) -29
(c) -21
(d) 51
(e) None of these

4.3 #62. Find the minimum value of the quadratic function $f(x) = 4x^2 - 8x + 3$.

(a) -5
(b) -1
(c) 3
(d) 15
(e) 35

4.3 #81. Suppose that the manufacturer of a gas clothes dryer has found that when the unit price is p dollars the revenue R (in dollars) is $R(p) = -4p^2 + 4000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

(a) $4000
(b) $1,000,000
(c) $500
(d) $3,000,000
(e) None of these

4.3 E. A store selling calculators has found that, when the calculators are sold at a price of p dollars per unit, the revenue R (in dollars) as a function of the price p is $R(p) = -750p^2 + 15000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

(a) $10
(b) $100
(c) $60,000
(d) $75,000
(e) None of these

Answer Key

4.1 #29. (d)

4.1 #37c. (b)

4.3 A. (b)

4.3 #42 (c)

4.3 B. (d)

4.3 C. (a)

4.3 D. (b)

4.3 #55. (c)

4.3 #61. (c)

4.3 #62. (b)

4.3 #81. (b)

4.3 E. (d)

Solutions

4.3 A. If $f(x)$ is a quadratic function whose graph has the vertex (h, k), which one is the correct form of the function?

Solution: (b) $f(x) = a(x-h)^2 + k$

4.3 B. Find the vertex of the quadratic function $f(x) = 2x^2 - 4x + 9$.

Solution: (d) The text gives this formula: the x-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is $x = -\frac{b}{2a}$. In this example, $a = 2$ and $b = -4$, so the vertex occurs at $x = -\frac{-4}{2 \cdot 2} = 1$. Then $f(1) = 2 - 4 + 9 = 7$ gives the y-coordinate.

If you forget the formula, you can always complete the square:
$f(x) = 2x^2 - 4x + 9 = 2(x^2 - 2x) + 9 = 2(x^2 - 2x + 1) + 9 - 2 = 2(x-1)^2 + 7$
so $h = 1$ and $k = 7$ and the vertex is $(1, 7)$.

4.3 C. Find the axis of symmetry of the graph of $f(x) = 4x^2 - 8x + 3$.

Solution: (a) The axis of symmetry passes through the vertex, which has x-coordinate $-\frac{-8}{2 \cdot 4} = 1$. The axis of symmetry is the line $x = 1$.

Again, if you forget the formula, complete the square:
$f(x) = 4x^2 - 8x + 3 = 4(x^2 - 2x) + 3 = 4(x^2 - 2x + 1) + 3 - 4 = 4(x-1)^2 - 1$
This shows that the vertex is at $(1, -1)$.

4.3 D. Let $f(x) = 4x^2 - 8x + 3$. Find the x and y-intercepts, if any.

Solution: (b) Since $f(0) = 3$, the y-intercept is $(0, 3)$.

To find the x-intercept, solve $4x^2 - 8x + 3 = 0$. This can be factored as $4x^2 - 8x + 3 = (2x-1)(2x-3)$, so $2x - 1 = 0$ or $2x - 3 = 0$, giving the x-intercepts $(\frac{3}{2}, 0)$ and $(\frac{1}{2}, 0)$.

4.3 #62. Find the minimum value of the quadratic function $f(x) = 4x^2 - 8x + 3$.

Solution: (b) The minimum value occurs at the vertex, which has x-coordinate $-\frac{-8}{2\cdot 4} = 1$. Then $f(1) = -1$ is the minimum height.

4.3 E. A store selling calculators has found that, when the calculators are sold at a price of p dollars per unit, the revenue R (in dollars) as a function of the price p is $R(p) = -750p^2 + 15000p$. What is the largest possible revenue? That is, find the maximum value of the revenue function.

Solution: (d) The graph is a parabola, opening down, so to find the maximum value we need to find the y-coordinate of the vertex. We get $x = -\frac{b}{2a} = -\frac{15000}{2\cdot(-750)} = -\frac{15000}{-1500} = 10$. The maximum revenue is $R(10) = -750(10)^2 + 15000(10) = -75000 + 150000 = 75000$.

Chapter 5

Polynomial and Rational Functions

Section summaries

Section 5.1 Polynomial Functions

The **general form of a polynomial function** is $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$. The **degree** of $f(x)$ is the largest exponent in the formula. Linear functions $f(x) = mx + b$ and quadratic functions $f(x) = ax^2 + bx + c$ are the simplest cases. If $|x|$ is large, then the term $a_n x^n$ is much larger than the others, so the "big picture" of $f(x)$ is that its graph follows the pattern of x^n, flipped over if a_n is negative.

The number of different x-intercepts of a polynomial of degree n is at most n, because a polynomial equation of degree n has at most n roots. The same is true of any horizontal line–the graph of a polynomial of degree n can cross the line at most n times. This means that the graph has at most $n - 1$ "turning points" (see the discussion on the top of page 321), and this helps you in graphing.

Finally, a polynomial has two types of behavior at an x-intercept. It may cross the x-axis, like $y = x^3$, or it may just touch the x-axis, like $y = x^2$. If you can factor the function completely, you can tell whether it crosses or touches by looking at the exponent of the factor that corresponds to the root you are interested in. If the exponent is odd, the graph will cross the axis because the y-values will change sign, but if the exponent is even, the graph will just touch the axis and stay on the same side.

You should review the summaries on pages 336 and 338 very carefully.

Review problems: p340 #31,37,45,55,63,71,75

Section 5.2 Properties of Rational Functions

We have been building up to more and more complicated functions. This section deals with some basic properties of functions of the form $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials. These are called **rational** functions. The functions $f(x) = \dfrac{1}{x}$ and $f(x) = \dfrac{1}{x^2}$ are two familiar examples.

Just as $f(x) = \frac{1}{x}$ has a graph that is asymptotic to the axes, a general rational function can have horizontal and vertical asymptotes. It may or may not cross the x-axis.

x-intercepts: The only way $f(x)$ can be zero is if the numerator is zero, so you can find the x-intercepts by setting the numerator equal to zero, and solving the equation $p(x) = 0$.

vertical asymptotes: These can be found by looking at the values of x at which $f(x)$ is not defined (because of division by zero). You just need to set the denominator equal to zero, and solve the equation $q(x) = 0$. Note: you must first make sure that the numerator and denominator do not have any common factors.

horizontal asymptotes: if $|x|$ is large, the function

$$f(x) = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + a_1 x + a_0}$$

behaves like $y = \frac{a_m x^m}{b_n x^n}$, just like we learned for polynomials. If the numerator and denominator have the same degree, this reduces to a constant, and gives the equation of the asymptote. If the denominator has larger degree than the numerator, then $y = 0$ is a horizontal asymptote. If the numerator has larger degree than the denominator, then there is no horizontal asymptote (you will not be tested on oblique asymptotes). To just find the horizontal asymptotes you do *not* need to use long division to write $f(x)$ as a "mixed fraction."

Review problems: p352 #15,21,27,35,39,41,45,47

Section 5.4 Polynomial and Rational Inequalities

The method of solution is given on page 370. In terms of the graph of a polynomial or rational function, we need to determine when the graph is above or below the x-axis. We first decide where the graph can change from positive to negative, and from negative to positive. Then if we know that we have found an interval on which the graph cannot change sign, it is enough to test one point in the interval. Just be very careful in working these problems.

Review problems: p373 #5,13,25,29,33,51

Sample Questions

5.1 #38. Form a polynomial function of degree 3 with zeros -2, 2, 3.

(a) $f(x) = (x^2 - 4)(x - 3)^2$

(b) $f(x) = (x^2 - 4)(x + 3)^2$

(c) $f(x) = (x^2 - 4)(x - 3)$

(d) $f(x) = (x^2 - 4)(x + 3)$

(e) None of these

5.1 A. The polynomial function $f(x)$ has a zero at $x = 2$ with multiplicity 3. We know that

(a) since 3 is an odd number, the graph touches but does not cross the x-axis.

(b) since 3 is an odd number, the graph crosses the x-axis.

(c) since 2 is an even number, the graph touches but does not cross the x-axis.

(d) since 2 is an even number, the graph crosses the x-axis.

(e) none of these occurs.

5.1 B. Find all the zeros and their multiplicities for the polynomial $p(x) = 11x(x-1)^5(x+6)$.

(a) -1 is a zero of multiplicity 5; 6 is a zero of multiplicity 1; 0 is a zero of multiplicity 1

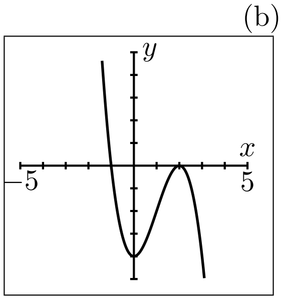
(b) 1 is a zero of multiplicity 5; -6 is a zero of multiplicity 1; 0 is a zero of multiplicity 1

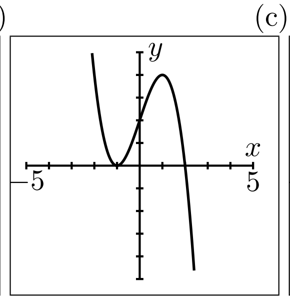
(c) -1 is a zero of multiplicity 5 and 6 is a zero of multiplicity 1

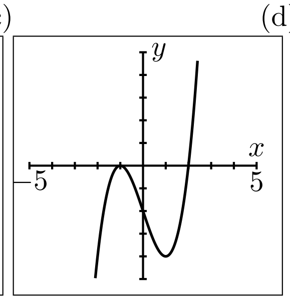
(d) 1 is a zero of multiplicity 5; -6 is a zero of multiplicity 1.

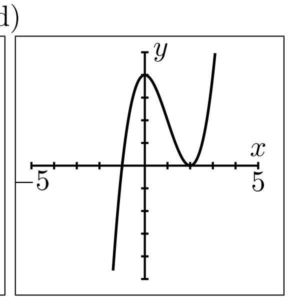
(e) None of these

5.1 C. Which of the following is the graph of $f(x) = (x + 1)^2(x - 2)$?

(a)


(b)


(c)


(d)


5.1 D. The function $f(x) = x^2(x - 2)(x + 3)^2$ has

(a) one zero of multiplicity 1 and one zero of multiplicity 2.

(b) one zero of multiplicity 1 and two zeros of multiplicity 2.

(c) one zero of multiplicity 1 and three zeros of multiplicity 2.

(d) three zeros of multiplicity 1 and one zero of multiplicity 2.

(e) None of these

5.1 E. Which one of these functions might have the given graph?

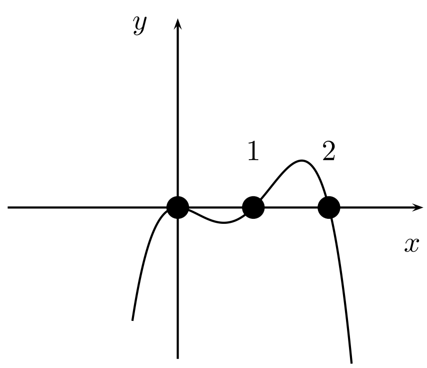
(a) $f(x) = x(x-1)(x-2)^2$

(b) $f(x) = -x(x-1)(x-2)$

(c) $f(x) = x(x-1)(x-2)$

(d) $f(x) = x^2(x-1)(x-2)$

(e) $f(x) = -x^2(x-1)(x-2)$



5.2 A. Find the domain of the function $f(x) = \dfrac{x-2}{x+1}$.

(a) All real numbers except -1

(b) All real numbers except 1

(c) All real numbers except -2

(d) All real numbers except 2

(e) None of these

5.2 B. What is the domain of the function G defined by $G(x) = \dfrac{x+4}{x^3-4x}$?

(a) all reals except -4

(b) all reals except $2, -2$

(c) all reals except $0, 2, -2$

(d) $\{0, 2, -2\}$

(e) $\{0, 2, -2, -4\}$

5.2 C. The graph of $y = \dfrac{1}{(x-4)^2}$ looks like that of $y = \dfrac{1}{x^2}$ but is shifted

(a) left 4 units

(b) right 4 units

(c) down 4 units

(d) up 4 units

(e) None of these

5.2 D. Find the vertical asymptotes of the graph of $f(x) = \dfrac{x^2 - 3x}{x^2 - 2x - 8}$.

(a) $x = -4$ and $x = 2$

(b) $x = 4$ and $x = -2$

(c) $x = 0$ and $x = 3$

(d) $x = 4$, $x = -2$, $x = 0$ and $x = 3$

(e) None of these

5.2 E. The line $x = 4$ is a vertical asymptote of the graph of which of the following functions?

(a) $f(x) = x - 4$

(b) $f(x) = \dfrac{2x - 8}{x - 4}$

(c) $f(x) = \dfrac{1}{x^2 - 16}$

(d) $f(x) = \dfrac{x - 4}{x + 3}$

(e) $f(x) = \dfrac{4x + 1}{x + 2}$

5.2 F. Find the x-intercepts and vertical asymptotes of the graph of $f(x) = \dfrac{x^2 - 3x}{x^2 - 2x - 8}$

(a) x-intercepts $(4, 0), (3, 0), (-2, 0), (0, 0)$, vertical asymptotes $x = 4, x = 3, x = -2, x = 0$

(b) x-intercepts $(-4, 0), (2, 0)$, vertical asymptotes $x = 0, x = 3$

(c) x-intercepts $(4, 0), (-2, 0)$, vertical asymptotes $x = 0, x = 3$

(d) x-intercepts $(0, 0), (3, 0)$, vertical asymptotes $x = 4, x = -2$

(e) x-intercepts $(0, 0), (3, 0)$, vertical asymptotes $x = -4, x = 2$

5.2 G. Find the vertical asymptotes of $f(x) = \dfrac{(x-1)(x+2)(x-3)}{x(x-4)^2}$.

(a) $x = 1, -2, 3$

(b) $x = 0, 1, -2, 3, 4$

(c) $x = 0, 4$

(d) $x = 4$

(e) None of these

5.2 H. Find the horizontal asymptote of the graph of $f(x) = \dfrac{x - 3}{5x + 2}$.

(a) $y = \frac{1}{5}$

(b) $y = -\frac{3}{2}$

(c) $x = -\frac{2}{5}$

(d) $x = -\frac{1}{3}$

(e) None of these

5.2 K. Find the horizontal asymptote for the graph of $f(x) = \dfrac{5x-1}{2x+3}$.

(a) $x = -\frac{3}{2}$
(b) $x = \frac{5}{2}$
(c) $y = -\frac{3}{2}$
(d) $y = \frac{5}{2}$
(e) None of these

5.2 #48. Find the horizontal asymptote (if any) of $f(x) = \dfrac{-2x^2+1}{2x^3+4x^2}$.

(a) $y = -2$
(b) $y = -1$
(c) $y = 1$
(d) $y = 0$
(e) None of these

5.2 L. Which one of these functions does **not** have a horizontal asymptote?

(a) $f(x) = \dfrac{2}{3x-5}$
(b) $f(x) = \dfrac{2x^2+1}{3x-5}$
(c) $f(x) = \dfrac{2x^2+1}{3x^2-5}$
(d) $f(x) = \dfrac{2x}{3x^2-5}$
(e) $f(x) = 2 + \dfrac{6}{3x^2-5}$

5.2 M. Find the asymptotes of the following function. $f(x) = \dfrac{3x^2-3x}{x^2+x-12}$

(a) The horizontal asymptote is $y = 0$; the vertical asymptotes are $x = -4$ and $x = 3$.
(b) The horizontal asymptote is $y = 3$; the vertical asymptotes are $x = 4$ and $x = -3$.
(c) The horizontal asymptote is $y = 3$; the vertical asymptotes are $x = -4$ and $x = 3$.
(d) The horizontal asymptote is $y = -4$; the vertical asymptote is $x = 3$.
(e) None of these

5.4 #5. Solve the inequality $x^3 - 4x^2 > 0$.

(a) $(4, \infty)$
(b) $(0, \infty)$
(c) $(-\infty, 4)$
(d) $(-\infty, 0)$
(e) None of these

5.4 #13. Solve the inequality $(x-1)(x-2)(x-3) \leq 0$.

(a) $(-\infty, 1] \cup [2, \infty]$

(b) $(-\infty, 2] \cup [3, \infty]$

(c) $(-\infty, 1] \cup [2, 3]$

(d) $[1, 2] \cup [3, \infty)$

(e) None of these

Answer Key

5.1 #38. (c)

5.1 A. (b)

5.1 B. (b)

5.1 C. (c)

5.1 D. (b)

5.1 E. (e)

5.2 A. (a)

5.2 B. (c)

5.2 C. (b)

5.2 D. (b)

5.2 E. (c)

5.2 F. (d)

5.2 G. (c)

5.2 H. (a)

5.2 K. (d)

5.2 #48. (d)

5.2 L. (b)

5.2 M. (c)

5.4 #5. (a)

5.4 #13. (c)

Solutions

5.1 #38. Form a polynomial function of degree 3 with zeros -2, 2, 3.

Solution: (c) The polynomial should include a linear factor for each zero, so we should use $f(x) = (x+2)(x-2)(x-3) = (x^2-4)(x-3)$.

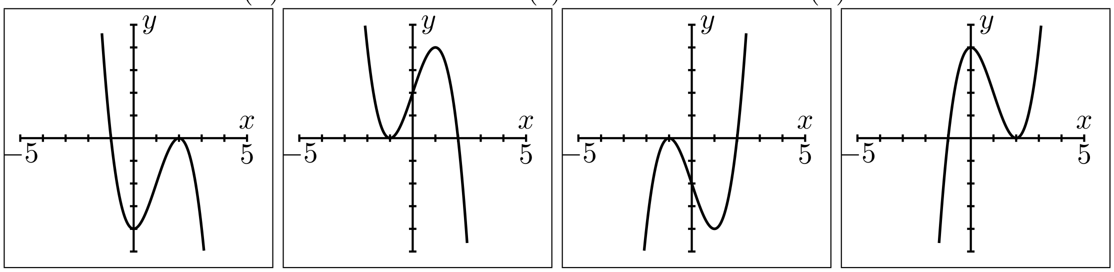
5.1 A. The polynomial function $f(x)$ has a zero at $x = 2$ with multiplicity 3. Answer: (b) Since the multiplicity of the zero is 3, which is an odd number, the graph crosses the x-axis.

5.1 B. Find all the zeros and their multiplicities for the polynomial $p(x) = 11x(x-1)^5(x+6)$.

Solution: (b) Setting the factors x, $x-1$, and $x+6$ each equal zero shows that the zeros are 0, 1, and -6. The next step is to look at the exponent of each factor: x has exponent 1, so the multiplicity of the root 0 is 1; $x-1$ has exponent 5, so the multiplicity of the root 1 is 5; $x+6$ has exponent 1, so the multiplicity of the root -6 is 1.

5.1 C. Which of the following is the graph of $f(x) = (x+1)^2(x-2)$?

(a) (b) (c) (d)



Solution: (c) The function has roots -1 and 2, so these must be x-intercepts on the graph. All four graphs pass this test, so we have to look at it more deeply. The multiplicity of -1 is 2, so the graph should only touch the axis at $x = -1$. Since the multiplicity of 2 is just 1, the graph should cross the axis at $x = 2$. This eliminates answers (a) and (d). We could plot one more point: since $f(0) = -2$, the y=intercept must be -2, and this eliminates answer (b).

5.1 D. The function $f(x) = x^2(x-2)(x+3)^2$ has
Answer: (b) one zero of multiplicity one and two zeros of multiplicity two.
Solution: $x = 0$ has multiplicity 2; $x = 2$ has multiplicity 1; $x = -3$ has multiplicity 2.

5.1 E. Which one of these functions might have the given graph?

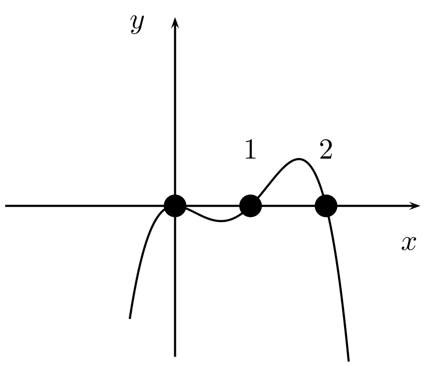
(a) $f(x) = x(x-1)(x-2)^2$

(b) $f(x) = -x(x-1)(x-2)$

(c) $f(x) = x(x-1)(x-2)$

(d) $f(x) = x^2(x-1)(x-2)$

(e) $f(x) = -x^2(x-1)(x-2)$



Solution: (e) The roots must be $x = 0$, $x = 1$, and $x = 2$, and this is true for every formula. Since that graph touches the x-axis at $x = 0$ but does not cross, the root $x = 0$ must have even multiplicity. The graph crosses the x-axis at $x = 1$ and $x = 2$, so these roots must have odd multiplicity. This eliminates the formulas in (a), (b), and (c).

Finally, to tell the difference between (d) and (e), check the value of the function at $x = -2$. In (d) we get $f(-2) = (-2)^2(-2-1)(-2-2) = 48$, so this doesn't agree with the graph, so the answer must be (e).

Alternatively, you can see that for large values of x the formula in (d) behaves like $f(x) = x^4$, while the formula in (e) behaves like $f(x) = -x^4$. The behavior of the graph is like $f(x) = -x^4$, so the answer must be (e).

5.2 A. Find the domain of the function $f(x) = \frac{x-2}{x+1}$.

Solution: (a) Set the denominator equal to 0. Exclude $x = -1$ to avoid division by 0.

5.2 B. What is the domain of the function G defined by $G(x) = \frac{x+4}{x^3-4x}$?

Solution: (c) Set $x^3 - 4x = 0$. This gives $x(x^2 - 4) = 0$, or $x(x-2)(x+2) = 0$. The numbers 0, 2, and -2 must be excluded. Answer: all real numbers except $0, 2, -2$.

5.2 C. The graph of $y = \frac{1}{(x-4)^2}$ looks like that of $y = \frac{1}{x^2}$ but is shifted

Solution: (b) right 4 units. One way to remember which way it is shifted is to observe that the graph of $y = \frac{1}{x^2}$ has a vertical asymptote at $x = 0$, while the graph of $y = \frac{1}{(x-4)^2}$ has a vertical asymptote at $x = 4$.

5.2 D. Find the vertical asymptotes of the graph of $f(x) = \frac{x^2-3x}{x^2-2x-8}$.

Solution: (b) Set the denominator equal to 0. $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ The vertical asymptotes occur at $x = 4$ and $x = -2$.

5.2 E. The line $x = 4$ is a vertical asymptote of the graph of which of the following functions?
(a) $f(x) = x - 4$ (b) $f(x) = \frac{2x-8}{x-4}$ (c) $f(x) = \frac{1}{x^2-16}$ (d) $f(x) = \frac{x-4}{x+3}$ (d) $f(x) = \frac{4x+1}{x+2}$

Solution: (c) To have a vertical asymptote at $x = 4$, the denominator must have a factor of $x - 4$. This seems to be true for both (b) and (c). Actually, $f(x) = \frac{2x-8}{x-4} = \frac{2(x-4)}{x-4} = 2$ for all values except $x = 4$. In the function in (b), the graph is a horizontal line at $y = 2$, with the point $(4, 2)$ missing, so it has no vertical asymptote. The correct answer is that only $f(x) = \frac{1}{x^2-16}$ has a graph with a vertical asymptote at $x = 4$.

5.2 F. Find the x-intercepts and vertical asymptotes of the graph of $f(x) = \frac{x^2-3x}{x^2-2x-8}$

Solution: (d) x-intercepts $(0, 0), (3, 0)$, vertical asymptotes $x = 4, x = -2$ Set the numerator equal to 0 to find the x-intercepts. $x^2 - 3x = 0$ $x(x-3) = 0$ $x = 0$ and $x = 3$.
Set the denominator equal to 0 to find the vertical asymptotes. $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4$ and $x = -2$

5.2 G. Find the vertical asymptotes of $f(x) = \frac{(x-1)(x+2)(x-3)}{x(x-4)^2}$.

Solution: (c) Set the denominator equal to 0 to get $x = 0$ and $x = 4$.

5.2 H. Find the horizontal asymptote of the graph of $f(x) = \frac{x-3}{5x+2}$.

Solution: (a) Multiplying both the numerator and denominator by $\frac{1}{x}$ gives $f(x) = \frac{1-\frac{3}{x}}{5+\frac{2}{x}}$, and in this form we see that as x increases the function gets closer and closer to $\frac{1}{5}$.
The shortcut is to remember that the highest powers of x dictate the behavior, so $f(x)$ will behave like $\frac{x}{5x} = \frac{1}{5}$ for large values of x.

5.2 K. Find the horizontal asymptote for the graph of $f(x) = \frac{5x-1}{2x+3}$.

Solution: (d) For large values of x the function behaves like $f(x) = \frac{5x}{2x}$, so the horizontal asymptote is $y = \frac{5}{2}$.

5.2 #48. Find the horizontal asymptote (if any) of $f(x) = \frac{-2x^2+1}{2x^3+4x^2}$.

Solution: (d) For large values of x, the function behaves like $f(x) = \frac{-2x^2}{2x^3} = -\frac{1}{x}$, so $y = 0$ is a horizontal asymptote. Remember that if the denominator has higher degree than the numerator, then $y = 0$ will be a horizontal asymptote.

5.2 L. Which one of these functions does **not** have a horizontal asymptote?
(a) $f(x) = \frac{2}{3x-5}$ (b) $f(x) = \frac{2x^2+1}{3x-5}$ (c) $f(x) = \frac{2x^2+1}{3x^2-5}$ (d) $f(x) = \frac{2x}{3x^2-5}$ (e) $f(x) = 2 + \frac{6}{3x^2-5}$

Solution: (b) If the degree of the numerator is larger than the degree of the denominator, then there is no horizontal asymptote. This happens for (b).

5.2 M. Find the asymptotes of the following function. $f(x) = \frac{3x^2-3x}{x^2+x-12}$

Solution: (c) For large values of x, the function behaves like $f(x) = \frac{3x^2}{x^2}$, so $y = 3$ is a horizontal asymptote.

To find the vertical asymptotes, solve $x^2 + x - 12 = 0$. $(x+4)(x-3) = 0$ The vertical asymptotes are $x = -4$ and $x = 3$.

Chapter 6

Exponential and Logarithmic Functions

Section summaries

Section 6.1 Composite Functions

Some functions are constructed in several steps, where each of the individual steps is a function. For example, you would evaluate $h(x) = (2x+3)^4$ by first computing $g(x) = 2x+3$ and then raising it to the 4th power. This is expressed mathematically by writing $h(x) = f(g(x))$, where $g(x) = 2x+3$ and $f(x) = x^4$. Here one formula is substituted into another, giving a **composite function**.

See page 402 for the definition of a composite function; see page 406 for an important application to calculus.

To find the domain of a composite function $f(g(x))$, start with the domain of $g(x)$. (The domain of $f(g(x))$ is always contained in the domain of g.) Then, depending on the formula you get for $f(g(x))$, you might need to exclude some more values.

Review problems: p407 #13,21,35,51,63,65

Section 6.2 Inverse Functions

The inverse of a function is like a "reverse look-up" function. Usually we use a formula $y = f(x)$ to find y when x is given. What about the reverse? Given y, how can you find x? This is the job of the inverse function. To give a definition of an inverse function, we use the notion of a composite function.

The function $g(x)$ is the **inverse** of $f(x)$ if $f(g(x)) = x$ and $g(f(x)) = x$, for all x. These equations say that g does exactly reverse of f. A good example to think of is $f(x) = \sqrt[3]{x}$ and $g(x) = x^3$.

When g is the inverse of f, we usually write $g = f^{-1}$, and read this as "g equals f inverse". See the box at the top of page 414 for the basic relationship used to define of f^{-1}:

$$f^{-1}(f(x)) = x \quad \text{and} \quad f(f^{-1}(x)) = x \ .$$

Not all functions have an inverse function. If two x-values produce the same y-value in the formula $y = f(x)$, then given y there is no unique way to recover x. In order to have an inverse, the graph of $y = f(x)$ must pass the horizontal line test (see page 411). If $y = f(x)$ passes the test, then we simply interchange x and y in the formula $y = f(x)$, and solve for y to get the formula for the inverse. See page 416 for this procedure that actually finds the formula for the inverse of a function.

A function and its inverse are closely connected. The domain of f^{-1} is the range of f; the range of f^{-1} is the domain of f (see page 413). The graph of f^{-1} is the mirror image of the graph of f, in the line $y = x$. This happens since if the point (x, y) is on the graph of f, then the symmetric point (y, x) must be on the graph of f^{-1} (see page 415).

Review problems: p420 #35,39,43,49,57,81

Section 6.3 Exponential Functions

An **exponential function** is one of the form

$$f(x) = a^x$$

where a is a positive real number and $a \neq 1$. (We will usually assume that $a > 1$.) The domain of an exponential function is the set of all real numbers. Its graph has the x-axis as a horizontal asymptote. The points $(0, 1)$, $(1, a)$, and $(-1, 1/a)$ are easy ones to plot. Note that if x increases by 1, then $f(x)$ is multiplied by a, since

$$f(x + 1) = a^{x+1} = a^x \cdot a = af(x)$$

(see the theorem on page 425).

You need to be familiar with the basic shape of an exponential function. See Figure 18 on page 426 for the graph of $y = 2^x$ and Figure 27 on page 430 for the graph of $y = e^x$. The base e is important because it makes calculations easier when doing calculus. (It is the one exponential function whose graph crosses the y-axis at a 45 degree angle, making the slope of the graph equal to 1 when $x = 0$.) For our class, the only thing you need to remember about e is its approximate value of 2.7 and the fact that the graph of $y = e^x$ lies between the graphs of $y = 2^x$ and $y = 3^x$.

Building on the basic shape of $y = a^x$, we can graph other functions in the family by using transformations (as we did in Section 3.5).

Review problems: p433 #21,25,43,53,65,71,89

Section 6.4 Log Functions

A **logarithmic function** (or **log function** for short) is one of the form

$$f(x) = \log_a(x)$$

where $a > 0$ and $a \neq 1$. If $a = 10$, we usually write $\log(x)$ instead of $\log_{10}(x)$. If $a = e$, we write $\ln(x)$ instead of $\log_e(x)$, and call this the **natural log** function.

The log function $\log_a(x)$ is defined to be the **inverse of the exponential function** a^x. First, this tells us the basic shape of the graph (see Figure 30 on page 440). It also guarantees that the graph has the y-axis as a vertical asymptote, and that the domain of $\log_a(x)$ is $(0, +\infty)$, the same as the range of a^x. Now, when finding the domain of a function, you not only need to watch out for division by zero, or the square root of a negative number, but also for the log of a negative number. All of these are undefined for real numbers.

To express the inverse relationship, we can say that $y = \log_a(x)$ if and only if $x = a^y$ (see the top of page 438). We also have the following equations, which summarize the inverse relationship (see the theorem at the top of page 451):

$$a^{\log_a(x)} = x \quad \text{and} \quad \log_a(a^x) = x \; .$$

These identities are important in solving equations that involve logs. For example, to solve the equation $\log_2(2x+1) = 3$ we need to simplify the left hand side. Since $2^{\log_2(2x+1)} = 2x+1$, the first step is to make both sides of the equation into an exponent with base 2, to get $2^{\log_2(2x+1)} = 2^3$, which simplifies to $2x + 1 = 8$. To solve the equation $\ln(e^{-2x}) = 8$, just note that the left hand side is equal to $-2x$, so the equation simplifies immediately to $-2x = 8$. To solve the equation $e^{2x+5} = 8$, we need to get rid of the base e on the left hand side. This is done by substituting both sides into the natural log function, to get $\ln(e^{2x+5}) = \ln(8)$, or simply $2x + 5 = \ln 8$.

Review problems: p446 #21,33,37,43,63,71,81,89,99

Section 6.5 Properties of Logarithms

Since logs represent exponents, they should behave like exponents. For example, if we write two numbers M and N in scientific notation as powers of 10, then to multiply M and N we only need to add the exponents. To find the square root of M, we only need to divide the exponent of M by 2. The crucial properties of logs are summarized in the following equations (see page 451 and 452).

$$\log_a(MN) = \log_a(M) + \log_a(N) \qquad \log_a(M/N) = \log_a(M) - \log_a(N) \qquad \log_a(M^r) = r\log_a(M)$$

There is also a formula to change the base: $\log_a(M) = \dfrac{\log_b(M)}{\log_b(a)}$. (See the section summary on page 456.)

Review problems: p457 #13,17,29,41,49,57,83,85

Section 6.6 Log and exponential equations

In this section, the properties of logarithms are used to solve various kinds of equations.

Review problems: p463 #17,31,35,77,81

Sample Questions

6.1 #11a. Let $f(x) = 2x$ and $g(x) = 3x^2 + 1$. Find $(f \circ g)(4)$.

(a) 337
(b) 193
(c) 98
(d) $24x^3 + 8x$
(e) None of these

6.1 #11. Let $f(x) = 2x$ and $g(x) = 3x^2 + 1$. Find the composite function $(g \circ f)(x)$.

(a) $12x^2 + 1$
(b) $6x^2 + 2$
(c) $6x^2 + 1$
(d) $6x^3 + 2x$
(e) $6x^3 + 1$

6.1 #15a. Let $f(x) = \sqrt{x}$ and let $g(x) = 2x$. Find $(f \circ g)(4)$.

(a) $\sqrt{2}$
(b) $2\sqrt{2}$
(c) 4
(d) 16
(e) None of these

6.1 A. Let $f(x) = 2x^2 + 1$ and let $g(x) = x + 3$. Find the composite function $(f \circ g)(x)$.

(a) $2x^2 + 18$
(b) $2x^2 + 19$
(c) $2x^2 + 12x + 18$
(d) $2x^2 + 12x + 19$
(e) None of these

6.1 #31b. Let $f(x) = 3x + 1$ and $g(x) = x^2$. Find the composite function $(g \circ f)(x)$.

(a) $x^2 + 3x + 1$
(b) $9x^2 + 1$
(c) $3x^3 + x^2$
(d) $9x^2 + 6x + 1$
(e) None of these

6.1 Example 4a. Find the domain of $f \circ g$ if $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{4}{x-1}$.

(a) $\{x \mid x \neq \pm 1\}$
(b) $\{x \mid x \neq 1\}$
(c) $\{x \mid x \neq -1\}$
(d) $\{x \mid x \neq -2\}$
(e) None of these

6.1 #35a1. Let $f(x) = \dfrac{3}{x-1}$ and $g(x) = \dfrac{2}{x}$. Find the composite function $(f \circ g)(x)$.

(a) $\dfrac{6}{x^2 - x}$

(b) $\dfrac{2x-2}{3}$

(c) $3x$

(d) $\dfrac{3x}{2-x}$

(e) None of these

6.1 #35a2. Let $f(x) = \dfrac{3}{x-1}$ and $g(x) = \dfrac{2}{x}$. Find the domain of $f \circ g$.

(a) $\{x \mid x \neq 1, x \neq 0\}$

(b) $\{x \mid x \neq 1, x \neq 0, x \neq 2\}$

(c) $\{x \mid x \neq 2\}$

(d) $\{x \mid x \neq 2, x \neq 1\}$

(e) $\{x \mid x \neq 2, x \neq 0\}$

6.1 #36. Find the domain of $f \circ g$ if $f(x) = \dfrac{1}{x+3}$ and $g(x) = \dfrac{-2}{x}$.

(a) $\{x \mid x \neq -3\}$

(b) $\{x \mid x \neq 2/3\}$

(c) $\{x \mid x \neq 0 \text{ and } x \neq -2/3\}$

(d) $\{x \mid x \neq 0 \text{ and } x \neq 2/3\}$

(e) None of these

6.1 #61. If $f(x) = 2x^2 + 5$ and $g(x) = 3x + a$, find a so that the y-intercept of $f \circ g$ is 23.

(a) $a = 8$

(b) $a = -8$

(c) $a = \pm 3\sqrt{2}$

(d) $a = \pm 3$

(e) None of these

6.1 #63a. Find $f \circ g$ for $f(x) = ax + b$ and $g(x) = cx + d$.

(a) $(f \circ g)(x) = acx^2 + (b+c)x + bd$

(b) $(f \circ g)(x) = acx + bc + d$

(c) $(f \circ g)(x) = acx + ad + b$

(d) $(f \circ g)(x) = acx + b + d$

(e) None of these

6.1 B. If $f(x) = 3x^2 - 7$ and $g(x) = 2x + a$, find a so that the graph of $f \circ g$ crosses the y-axis at 5.

(a) $a = \pm 2$

(b) $a = \pm 2\sqrt{3}$

(c) $a = \pm 3$

(d) $a = \pm 5$

(e) None of these

6.2 A. If $f(x)$ has an inverse, and $(2, -\frac{1}{2})$ is on the graph of $f(x)$, then what point is on the graph of $f^{-1}(x)$?

(a) $(\frac{1}{2}, -2)$
(b) $(-2, -\frac{1}{2})$
(c) $(-2, \frac{1}{2})$
(d) $(-\frac{1}{2}, 2)$
(e) $(2, -\frac{1}{2})$

6.2 #49. The inverse of the function $f(x) = 4x + 2$ is

(a) $f^{-1}(x) = \dfrac{x+2}{4}$
(b) $f^{-1}(x) = \dfrac{x+4}{2}$
(c) $f^{-1}(x) = \frac{1}{4}x - \frac{1}{2}$
(d) $f^{-1}(x) = \frac{1}{2}x - \frac{1}{4}$
(e) None of these

6.2 B. The function $f(x) = \sqrt{x-2}$, for $x \geq 2$, is a one-to-one function. Find the inverse function f^{-1}.

(a) $f^{-1}(x) = x^2 + 2$, for $x \geq 0$
(b) $f^{-1}(x) = x^2 + 2$, for $x \geq 2$
(c) $f^{-1}(x) = x^2 + 2$, for all x
(d) $f^{-1}(x) = -\sqrt{x-2}$, for $x \geq 2$
(e) $f^{-1}(x) = \dfrac{1}{\sqrt{x-2}}$, for $x > 2$

6.2 #57a. The function $f(x) = \dfrac{1}{x-2}$ is a one-to-one function. Find the inverse function.

(a) $f^{-1}(x) = \dfrac{1}{x} - \frac{1}{2}$
(b) $f^{-1}(x) = \dfrac{1}{x} + 2$
(c) $f^{-1}(x) = \dfrac{1}{x-2}$
(d) $f^{-1}(x) = x - 2$
(e) None of these

6.2 #57b. Find the range of the function $f(x) = \dfrac{1}{x-2}$. (See the previous problem.)

(a) $\{y \mid y \neq 2\}$
(b) $\{y \mid y \neq -1/2\}$
(c) $\{y \mid y \neq 0\}$
(d) All real numbers
(e) None of these

6.2 #59a. The function $f(x) = \dfrac{2}{x+3}$ is a one-to-one function. Find the inverse function.

(a) $f^{-1}(x) = \dfrac{2}{x} - \frac{2}{3}$

(b) $f^{-1}(x) = \dfrac{2}{x} + \frac{2}{3}$

(c) $f^{-1}(x) = \dfrac{2}{x} - 3$

(d) $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

(e) None of these

6.2 #59b. Find the range of the function $f(x) = \dfrac{2}{x+3}$. (See the previous problem.)

(a) $\{y \mid y \neq 0\}$

(b) $\{y \mid y \neq 2/3\}$

(c) $\{y \mid y \neq 1/2\}$

(d) All real numbers

(e) None of these

6.2 #60. The function $f(x) = \dfrac{4}{2-x}$, for $x \neq 2$, is a one-to-one function. Find the inverse function f^{-1}.

(a) $f^{-1}(x) = 2 - \dfrac{4}{x}$

(b) $f^{-1}(x) = \dfrac{1}{2} - \dfrac{1}{4}x$

(c) $f^{-1}(x) = \dfrac{-4}{2-x}$

(d) $f^{-1}(x) = \dfrac{4}{x} - 2$

(e) None of these

6.2 #63a. The function $f(x) = \dfrac{2x}{3x-1}$ is a one-to-one function. Find the inverse f^{-1}.

(a) $f^{-1}(x) = \dfrac{3x-1}{2x}$

(b) $f^{-1}(x) = \dfrac{x}{3}$

(c) $f^{-1}(x) = \dfrac{x}{3x-2}$

(d) $f^{-1}(x) = \dfrac{x}{2-3x}$

(e) None of these

6.2 #63b. Find the range of the function $f(x) = \dfrac{2x}{3x-1}$. (See the previous problem.)

(a) all real numbers except $\frac{1}{3}$

(b) all real numbers except $-\frac{1}{3}$

(c) all real numbers except $\frac{2}{3}$

(d) all real numbers except $-\frac{2}{3}$

(e) all real numbers except 0

6.2 C. The function $f(x) = 3x - 2$ is a one-to-one function. Find the inverse function f^{-1}.

(a) $f^{-1}(x) = \dfrac{1}{3x-2}$

(b) $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

(c) $f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

(d) $f^{-1}(x) = \frac{1}{3}x + \frac{2}{3}$

(e) None of these

6.2 #94. The period T of a simple pendulum is $T = 2\pi\sqrt{\dfrac{x}{g}}$, where x is its length and g is a constant (the acceleration due to gravity). Solve for x as a function of T.

(a) $x = 2\pi\sqrt{\dfrac{T}{g}}$

(b) $x = \dfrac{gT}{2\pi}$

(c) $x = \dfrac{gT^2}{2\pi}$

(d) $x = \dfrac{gT^2}{4\pi^2}$

(e) None of these

6.3 A. Which answer describes the graph of the exponential function $f(x) = e^x$?

(a) The graph goes through $(0, e)$ and decreases as x increases.

(b) The graph goes through $(0, e)$ and increases as x increases.

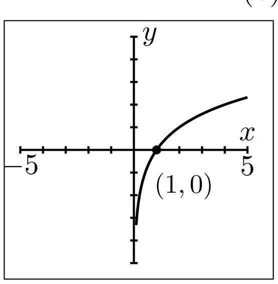
(c) The graph goes through $(0, 1)$ and decreases as x increases.

(d) The graph goes through $(0, 1)$ and increases as x increases.

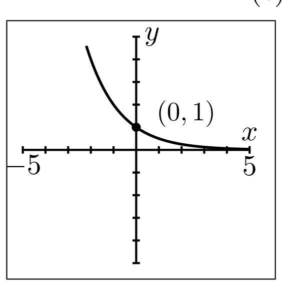
(e) The graph is a straight line through $(1, e)$.

6.3 B. Which of the following is the graph of $y = 2^x$?

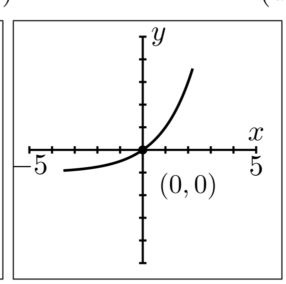
(a)



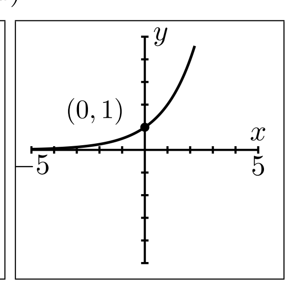
(b)



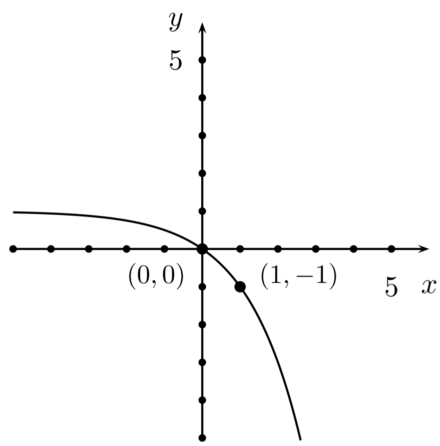
(c)



(d)



6.3 C. Which exponential function is represented by this graph?



(a) $f(x) = -2x^2 + x$

(b) $f(x) = 1 - 2^{-x}$

(c) $f(x) = 1 - 2^x$

(d) $f(x) = 1 + 2^x$

(e) $f(x) = 1 + e^x$

6.3 #43. Find the horizontal asymptote of $f(x) = 3^{-x} - 2$.

(a) $y = 2$

(b) $y = 0$

(c) $y = -2$

(d) $x = 0$

(e) $x = -2$

6.3 #52a. The horizontal asymptote of the graph of $y = e^x - 1$ is

(a) $y = -1$

(b) $y = 0$

(c) $y = 1$

(d) $y = e$

(e) There is no horizontal asymptote

6.3 #52b. The vertical asymptote of the graph of $y = e^x - 1$ is

(a) $x = -1$

(b) $x = 0$

(c) $x = 1$

(d) $x = e$

(e) There is no vertical asymptote

6.3 #55. Find the horizontal asymptote of the graph of $f(x) = 2 - e^{-x/2}$.

(a) $y = -2$

(b) $y = 0$

(c) $x = -2$

(d) $x = 0$

(e) None of these

6.3 D. Solve for x: $9^{2x} = 27$

(a) $x = \log_9 27$
(b) $x = \log_3 27$
(c) $x = \frac{4}{3}$
(d) $x = \frac{3}{4}$
(e) None of these

6.3 #61. Solve for x: $\left(\frac{1}{5}\right)^x = \frac{1}{25}$

(a) $x = -2$
(b) $x = -1/2$
(c) $x = 1/2$
(d) $x = 2$
(e) None of these

6.3 #63. Solve for x: $2^{2x-1} = 4$

(a) $x = 0$
(b) $x = -\frac{1}{2}$
(c) $x = 1$
(d) $x = \frac{3}{2}$
(e) There is no solution

6.3 E. Solve for x: $\sqrt{3}^{\,x+2} = \frac{1}{9}$

(a) $x = -4$
(b) $x = -5$
(c) $x = -6$
(d) $x = -3/2$
(e) None of these

6.3 #85. If $3^{-x} = 2$, what does 3^{2x} equal?

(a) 4
(b) -4
(c) $\frac{1}{4}$
(d) $-\frac{1}{4}$
(e) None of these

6.3 #86. If $5^{-x} = 3$, what does 5^{3x} equal?

(a) -9
(b) -1
(c) $-1/9$
(d) $1/9$
(e) None of these

6.4 A. Which answer describes the graph of the logarithmic function $f(x) = \ln x$?

(a) The graph goes through $(0, 1)$ and has $x = 0$ as a vertical asymptote.

(b) The graph goes through $(1, 0)$ and has $x = 0$ as a vertical asymptote.

(c) The graph goes through $(0, 1)$ and has $y = 0$ as a horizontal asymptote.

(d) The graph goes through $(1, 0)$ and has $y = 0$ as a horizontal asymptote.

(e) The graph is a straight line through $(0, 1)$ and $(e, 1)$.

6.4 B. List the properties of the graph of $y = \ln x$.

A: The graph has a vertical asymptote at $x = 1$.
B: The graph has a vertical asymptote at $x = 0$.
C: The graph goes through $(e, 0)$.
D: The graph goes through $(1, 0)$.
E: The graph has a horizontal asymptote.
F: The graph increases as x increases.

(a) A, C, and E

(b) A, D, and E

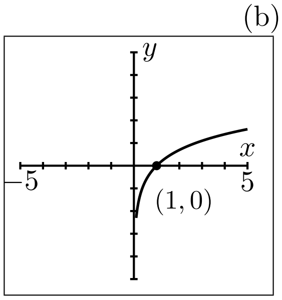
(c) A, C, and F

(d) B, D, and E

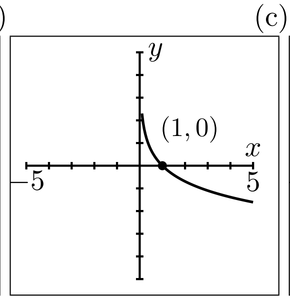
(e) B, D, and F

6.4 C. Which of the following is the graph of $y = \ln x$?

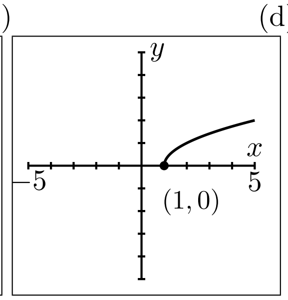
(a)



(b)



(c)



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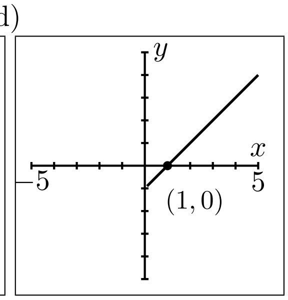


6.4 D. Which of the following pairs of functions are inverses of each other?

(a) $\ln(x)$ and 10^x

(b) $\log(x)$ and e^x

(c) $\ln(x)$ and e^x

(d) $\log_{2.7}(x)$ and e^x

(e) $\log_2(x)$ and $\left(\frac{1}{2}\right)^x$

6.4 E. If $f(x) = \log_3(x)$, what is $f^{-1}(x)$?

(a) $f^{-1}(x) = e^x$

(b) $f^{-1}(x) = 3^x$

(c) $f^{-1}(x) = -\log_3(x)$

(d) $f^{-1}(x) = \dfrac{1}{\log_3(x)}$

(e) $f^{-1}(x) = \log_{\frac{1}{3}}(x)$

6.4 #29. $\log_{\frac{1}{2}} 16 =$

(a) 8
(b) 4
(c) $\frac{1}{4}$
(d) $-\frac{1}{4}$
(e) -4

6.4 #33. $\log_{\sqrt{2}} 4 =$

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

6.4 #35. $\ln \sqrt{e} =$

(a) -1
(b) .5
(c) 1.359
(d) 2.718
(e) None of these

6.4 F. The domain of $f(x) = \log(1 - 5x)$ is

(a) $(\frac{1}{5}, \infty)$
(b) $[\frac{1}{5}, \infty)$
(c) $(-\infty, \frac{1}{5}]$
(d) $(-\infty, \frac{1}{5})$
(e) None of these

6.4 #43. The domain of $f(x) = \ln\left(\dfrac{1}{x+1}\right)$ is

(a) $\{x \mid x \geq -1\}$
(b) $\{x \mid x \neq -1\}$
(c) $\{x \mid x < -1\}$
(d) $\{x \mid x > -1\}$
(e) None of these

6.4 #82. Find the vertical asymptote of the graph of $f(x) = 2 - \log_3(x + 1)$.

(a) $x = -1$
(b) $x = 0$
(c) $x = 1$
(d) $y = 0$
(e) None of these

6.4 G. The equation $\log_\pi x = \frac{1}{2}$ can be written in exponential form as

(a) $x = \left(\frac{1}{2}\right)^\pi$
(b) $x = \pi^{1/2}$
(c) $x^\pi = \frac{1}{2}$
(d) $\pi = x^{1/2}$
(e) $\pi = \left(\frac{1}{2}\right)^x$

6.4 #89. Solve: $\log_2(2x+1) = 3$

(a) $x = 1$

(b) $x = 0$

(c) $x = 3$

(d) $x = 4$

(e) None of these

6.5 #14. $\log_6 4 + \log_6 9 =$

(a) 2

(b) 13/6

(c) $\log_6 13$

(d) $\log_6(4/9)$

(e) None of these

6.5 A. $(\log_2 6)(\log_6 8) =$

(a) 2

(b) 3

(c) $\log_6 4$

(d) $\log_2(4/3)$

(e) None of these

6.5 B. $(\log_3 6)(\log_6 9) =$

(a) $\log_6 3$

(b) $\log_3(3/2)$

(c) 2

(d) 3

(e) None of these

6.5 #29. If $\ln 2 = a$ and $\ln 3 = b$, then $\ln \sqrt[5]{6} =$

(a) $\frac{1}{5}ab$

(b) $\frac{1}{5}(a+b)$

(c) $5ab$

(d) $5(a+b)$

(e) None of these

6.5 #46. $\log\left(\dfrac{x^3\sqrt{x+1}}{(x-2)^2}\right) =$

(a) $3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)$

(b) $3\log x + \frac{1}{2}\log(x+1) + 2\log(x-2)$

(c) $3\log x + \log(x+1) - \log(x-2)$

(d) $3\log x + \log(x+1) + \log(x-2)$

(e) None of these

6.5 #49. $\ln\left(\dfrac{5x\sqrt{1+3x}}{(x-4)^3}\right) =$

(a) $5\ln x + \ln(1+3x) + \ln(x-4)$

(b) $5\ln x + \ln(1+3x) - \ln(x-4)$

(c) $\ln 5 + \ln x + \frac{1}{2}\ln(1+3x) - 3\ln(x-4)$

(d) $\ln 5 + \ln x + \frac{1}{2}\ln(1+3x) + 3\ln(x-4)$

(e) None of these

6.5 #72. $\log_\pi \sqrt{2} =$

(a) $\dfrac{1}{2\ln\pi}$

(b) $\dfrac{\ln 2}{\ln\pi}$

(c) $\dfrac{\ln 2}{2\ln\pi}$

(d) $\dfrac{\ln 2}{\ln\pi}$

(e) None of these

6.5 #83. Express y as a function of x: $\ln y = \ln x + \ln(x+1) + \ln C$

(a) $y = 2x + 1 + C$

(b) $y = Cx(x+1)$

(c) $y = Ce^{x(x+1)}$

(d) $y = e^{Cx(x+1)}$

(e) None of these

6.5 #85. Express y as a function of x (the constant C is positive). $\ln y = 3x + \ln C$

(a) $y = \ln(3x) + C$

(b) $y = Ce^{3x}$

(c) $y = C^{3x}$

(d) $y = e^{3x} + C$

(e) None of these

6.5 #87. Solve for y (the constant C is positive): $\ln(y-3) = -4x + \ln C$

(a) $y = 3 - \dfrac{4}{\ln x} + C$

(b) $y = 3 + C^{-4x}$

(c) $y = 3 + Ce^{-4x}$

(d) $y = 3 + e^{-4x} + C$

(e) None of these

6.6 A. Solve for x: $\ln(x+1)+\ln(x)=\ln(6)$

(a) $x=1/5$

(b) $x=-3$

(c) $x=-3$ or $x=2$

(d) $x=2$

(e) None of these

6.6 B. Solve for x: $\log_2(3x-1)=3$

(a) $x=\frac{7}{3}$

(b) $x=\frac{10}{3}$

(c) $x=\frac{8}{3}$

(d) $x=\frac{4}{3}$

(e) None of these

6.6 C. Solve for x: $2^{x+1}=6$

(a) $x=\ln 3$

(b) $x=\ln 4$

(c) $x=\ln 4-\ln 2$

(d) $x=\dfrac{\ln 3}{\ln 2}$

(e) None of these

6.6 D. Solve for x: $2^{2x+1}=\left(\frac{1}{2}\right)^x$

(a) $x=\frac{1}{3}$

(b) $x=0$

(c) $x=-\frac{1}{3}$

(d) $x=-1$

(e) None of these

6.6 E. Solve for x: $5^x=3^{1-2x}$

(a) $x=3/7$

(b) $x=11/7$

(c) $\dfrac{\ln 3}{\ln 5+2\ln 3}$

(d) $\dfrac{1}{\ln 5+2\ln 3}$

(e) None of these

Answer Key

6.1 #11a. (c)

6.1 #11. (a)

6.1 #15a. (b)

6.1 A. (d)

6.1 #31b. (d)

6.1 Example 4a. (a)

6.1 #35a1. (d)

6.1 #35a2. (e)

6.1 #36. (d)

6.1 #61. (d)

6.1 #63a. (c)

6.1 B. (a)

6.2 A. (d)

6.2 #49. (c)

6.2 B. (a)

6.2 #57a. (b)

6.2 #57b. (c)

6.2 #59a. (c)

6.2 #59b. (a)

6.2 #60. (a)

6.2 #63a. (c)

6.2 #63b. (c)

6.2 C. (d)

6.2 #94. (d)

6.3 A. (d)

6.3 B. (d)

6.3 C. (c)

6.3 #43. (c)

6.3 #52a. (a)

6.3 #52b. (e)

6.3 #55. (e)

6.3 D. (d)

6.3 #61. (d)

6.3 #63. (d)

6.3 E. (c)

6.3 #85. (c)

6.3 #86. (e)

6.4 A. (b)

6.4 B. (e)

6.4 C. (a)

6.4 D. (c)

6.4 E. (b)

6.4 #29. (e)

6.4 #33. (e)

6.4 #35. (b)

6.4 F. (d)

6.4 #43. (d)

6.4 #82. (a)

6.4 G. (b)

6.4 #89. (e)

6.5 #14. (a)

6.5 A. (b)

6.5 B. (c)

6.5 #29. (b)

6.5 #46. (a)

6.5 #49. (c)

6.5 #72. (c)

6.5 #83. (b)

6.5 #85. (b)

6.5 #87. (c)

6.6 A. (d)

6.6 B. (e)

6.6 C. (d)

6.6 D. (c)

6.6 E. (c)

Solutions

6.1 A. Let $f(x) = 2x^2 + 1$ and let $g(x) = x + 3$. Find the composite function $(f \circ g)(x)$.

Solution: (d) Write the formula for $f(x)$ in this way: $f(\quad) = 2(\quad)^2 + 1$. Then you have $(f \circ g)(x) = f(g(x)) = 2g(x)^2 + 1 = 2(x+3)^2 + 1 = 2(x^2+6x+9)+1 = 2x^2+12x+19$.

6.1 #36. Find the domain of $f \circ g$ if $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{-2}{x}$.

Solution: (d) $\{x \mid x \neq 0 \text{ and } x \neq 2/3\}$ First, you must exclude $x = 0$, since it is not in the domain of the first function $g(x)$. If you compute the composite function $f(g(x))$, you get $f(g(x)) = \dfrac{1}{-\frac{2}{x} + 3} = \frac{x}{-2+3x}$, so you must *also* exclude $x = 2/3$ (found by setting the denominator $-2 + 3x$ equal to 0).

6.1 B. If $f(x) = 3x^2 - 7$ and $g(x) = 2x + a$, find a so that the graph of $f \circ g$ crosses the y-axis at 5.

Solution: (a) One method of solution is to compute the composite function $f(g(x))$. You get $f(g(x)) = 3(2x + a)^2 - 7 = 3(4x^2 + 4ax + a^2) - 7 = 12x^2 + 12ax + 3a^2 - 7$. The problem asks you to find a so that the y-intercept is 5. Since the y-intercept is $3a^2 - 7$, you need to solve $3a^2 - 7 = 5$. You get $3a^2 = 12$, so $a = \pm 2$.

Another method of solution is to find the y-intercept in two steps. The first is by substituting $x = 0$ into $g(x)$. You get $g(0) = a$, and then, as the second step, you get $f(g(0)) = 3a^2 - 7$. The answer $a = \pm 2$ again comes from the solution of the equation $3a^2 - 7 = 5$.

6.2 A. If $f(x)$ has an inverse, and $(2, -\frac{1}{2})$ is on the graph of $f(x)$, then what point is on the graph of $f^{-1}(x)$?

Solution: (d) $(-\frac{1}{2}, 2)$ If (a, b) is on the graph of $f(x)$, then (b, a) is on the graph of $f^{-1}(x)$.

6.2 B. The function $f(x) = \sqrt{x - 2}$, for $x \geq 2$, is a one-to-one function. Find the inverse function f^{-1}.

Solution: (a) $f^{-1}(x) = x^2 + 2$, for $x \geq 0$.
Step 1. Write the function in the form $y = \sqrt{x - 2}$.
Step 2. Interchange x and y to get $x = \sqrt{y - 2}$.
Step 3. Solve for y in terms of x. $x = \sqrt{y - 2}$ $x^2 = (\sqrt{y - 2})^2 = y - 2$ $y = x^2 + 2$

To find the domain of $f^{-1}(x)$, it may be easiest to find the range of $f(x)$. Since $x \geq 2$, this includes the square root of every number ≥ 0, so the range of $f(x)$ is $\{y \mid y \geq 0\}$. This means that the domain of $f^{-1}(x)$ is $\{x \mid x \geq 0\}$, so the solution is the one given above.

6.2 #60. The function $f(x) = \frac{4}{2-x}$, for $x \neq 2$, is a one-to-one function. Find the inverse function f^{-1}.

Solution: (a) $f^{-1}(x) = 2 - \frac{4}{x}$ Write $y = \frac{4}{2-x}$, then exchange x and y and solve for y.
$x = \frac{4}{2-y}$ $(2-y)x = 4$ $2x - yx = 4$ $-yx = 4 - 2x$ $y = \frac{4-2x}{-x}$ $y = \frac{2x-4}{x} = 2 - \frac{4}{x}$

6.2 C. The function $f(x) = 3x - 2$ is a one-to-one function. Find the inverse function f^{-1}.
Solution: (d) $f^{-1}(x) = \frac{1}{3}x + \frac{2}{3}$ $y = 3x - 2$ $x = 3y - 2$ $x + 2 = 3y$ $y = \frac{1}{3}x + \frac{2}{3}$

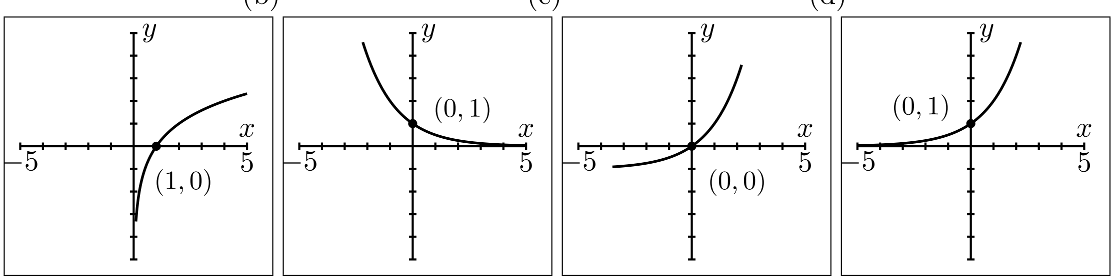
6.2 #94. The period T of a simple pendulum is $T = 2\pi\sqrt{\frac{x}{g}}$, where x is its length and g is a constant (the acceleration due to gravity). Solve for x as a function of T.

Solution: (d) $T = 2\pi\sqrt{\frac{x}{g}}$, $T^2 = 4\pi^2\left(\sqrt{\frac{x}{g}}\right)^2$ $T^2 = 4\pi^2\frac{x}{g}$ $gT^2 = 4\pi^2 x$
Answer: $x = \frac{gT^2}{4\pi^2}$

6.3 A. Which answer describes the graph of the exponential function $f(x) = e^x$?
(a) The graph goes through $(0, e)$ and decreases as x increases.
(b) The graph goes through $(0, e)$ and increases as x increases.
(c) The graph goes through $(0, 1)$ and decreases as x increases.
(d) The graph goes through $(0, 1)$ and increases as x increases.
(e) The graph is a straight line through $(1, e)$.

Solution: (d) When $x = 0$, we have $f(0) = e^0 = 1$, so the answer must be (c) or (d). As the exponent x increases, the values of e^x get larger and larger, so the y-values increase as x increases, and the answer must be (d).

6.3 B. Which of the following is the graph of $y = 2^x$?
(a) (b) (c) (d)



Solution: (d) The graph must go through $(0, 1)$, and must increase as x increases.

6.3 C. Which exponential function is represented by this graph? (see the original problem)

Solution: The first choice (a) $f(x) = -2x^2 + x$ is not an exponential function. The graph is decreasing, not increasing, so it cannot be (d) $f(x) = 1 + 2^x$ or (e) $f(x) = 1 + e^x$. We need to decide between (b) $f(x) = 1 - 2^{-x}$ and (c) $f(x) = 1 - 2^x$. Note that $(0, 0)$ and $(1, -1)$ are on the graph. Both (b) and (c) have $f(0) = 0$, but only (c) has $f(1) = -1$.

6.4 A. Which answer describes the graph of the logarithmic function $f(x) = \ln x$?

(a) The graph goes through $(0, 1)$ and has $x = 0$ as a vertical asymptote.
(b) The graph goes through $(1, 0)$ and has $x = 0$ as a vertical asymptote.
(c) The graph goes through $(0, 1)$ and has $y = 0$ as a horizontal asymptote.
(d) The graph goes through $(1, 0)$ and has $y = 0$ as a horizontal asymptote.
(e) The graph is a straight line through $(0, 1)$ and $(e, 1)$.

Solution: (b) The log function is the inverse of the exponential function, whose graph goes through $(0, 1)$, so its graph must go $(1, 0)$. It has a vertical asymptote, but no horizontal asymptote.

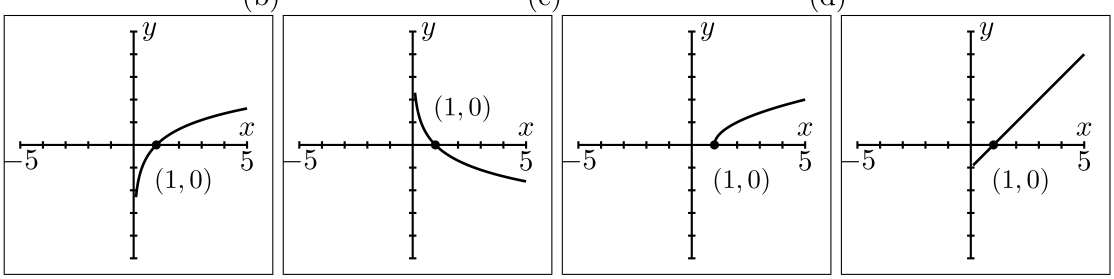
6.4 B. List the properties of the graph of $y = \ln x$.

A: The graph has a vertical asymptote at $x = 1$.
B: The graph has a vertical asymptote at $x = 0$.
C: The graph goes through $(e, 0)$.
D: The graph goes through $(1, 0)$.
E: The graph has a horizontal asymptote.
F: The graph increases as x increases.

Solution: (e) B, D, and F The graph does have a vertical asymptote, the y-axis, so B holds. Since $\ln(e) = 1$, condition C is false. The graph goes through $(1, 0)$ so D is true. The graph has no horizontal asyptote so E is false. Finally, F is true.

6.4 C. Which of the following is the graph of $y = \ln x$?

(a) (b) (c) (d)



Solution: (a) Use the properties of $y = \ln x$. The graph has a vertical asymptote at $x = 0$, goes through $(1, 0)$, and increases as x increases.

6.4 D. Which of the following pairs of functions are inverses of each other?

(a) $\ln(x)$ and 10^x (b) $\log(x)$ and e^x (c) $\ln(x)$ and e^x
(d) $\log_{2.7}(x)$ and e^x (e) $\log_2(x)$ and $\left(\frac{1}{2}\right)^x$

Solution: (c) The base must be the same in both functions. Since $\ln(x) = \log_e(x)$, the only pair for which this is true is (c).

6.4 E. If $f(x) = \log_3(x)$, what is $f^{-1}(x)$?

Solution: (b) $f^{-1}(x) = 3^x$ The inverse is an exponential function with the same base.

6.4 F. The domain of $f(x) = \log(1 - 5x)$ is

Solution: (d) $(-\infty, \frac{1}{5})$ The log function is only defined for positive values, so the domain is found by setting $1 - 5x > 0$. We get $x < \frac{1}{5}$.

6.4 #82. Find the vertical asymptote of the graph of $f(x) = 2 - \log_3(x+1)$.
Solution: (a) $x = -1$ The domain of $f(x)$ is $\{x \mid x+1 > 0\} = \{x \mid x > -1\}$.

6.4 G. The equation $\log_\pi x = \frac{1}{2}$ can be written in exponential form as
Solution: (b) To remove the $\log_\pi$, substitute both sides of the equation into the inverse function $g(x) = \pi^x$ of $f(x) = \log_\pi(x)$. This works because $\pi^{\log_\pi(x)} = x$ for all $x > 0$.
$\pi^{\log_\pi(x)} = \pi^{1/2}$ $x = \pi^{1/2}$

6.5 A. $(\log_2 6)(\log_6 8) =$

Solution: (b) Convert $\log_6 8$ to base 2 using the formula $\log_a M = \dfrac{\log_b M}{\log_b a}$ (see p455).
This gives $(\log_2 6)(\log_6 8) = (\log_2 6)\left(\dfrac{\log_2 8}{\log_2 6}\right) = \dfrac{(\log_2 6)(\log_2 8)}{\log_2 6} = \log_2 8 = 3$.

6.5 B. $(\log_3 6)(\log_6 9) =$

Solution: (c) $(\log_3 6)(\log_6 9) = (\log_3 6)\left(\dfrac{\log_3 9}{\log_3 6}\right) = \log_3 9 = 2$.

6.5 #46. $\log\left(\dfrac{x^3\sqrt{x+1}}{(x-2)^2}\right) =$

Solution: (a)

$$\begin{aligned}\log\left(\frac{x^3\sqrt{x+1}}{(x-2)^2}\right) &= \log(x^3) + \log(\sqrt{x+1}) - \log((x-2)^2)\\ &= 3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)\end{aligned}$$

6.5 #72. $\log_\pi \sqrt{2} =$

Solution: (c) $\log_\pi \sqrt{2} = \dfrac{\ln\sqrt{2}}{\ln \pi} = \dfrac{\frac{1}{2}\ln 2}{\ln \pi} = \dfrac{\ln 2}{2\ln \pi}$ by formula (9) on p455.

6.6 A. Solve for x: $\ln(x+1) + \ln(x) = \ln(6)$
Solution: (d) $\ln(x+1) + \ln(x) = \ln(6)$ $\ln(x+1)(x) = \ln(6)$ $x^2 + x = 6$
$x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3$ or $x = 2$
Since $\ln x$ is defined only for positive numbers, $x = -3$ cannot be a solution, and the correct answer is $x = 2$.

6.6 B. Solve for x: $\log_2(3x-1) = 3$
Solution: (e) To simplify by removing the term $\log_2$, use the inverse function 2^x.
$\log_2(3x-1) = 3$ $2^{\log_2(3x-1)} = 2^3$ $3x - 1 = 8$ $3x = 9$ $x = 3$

6.6 C. Solve for x: $2^{x+1} = 6$
Solution: (d) Since the answers are given in terms of ln, take the natural log of both sides. $2^{x+1} = 6$ $\ln(2^{x+1}) = \ln 6$ $(x+1)\ln 2 = \ln 6$ $(\ln 2)x + \ln 2 = \ln 6$
$(\ln 2)x = \ln 6 - \ln 2 = \ln \frac{6}{2} = \ln 3$ $x = \dfrac{\ln 3}{\ln 2}$

6.6 D. Solve for x: $2^{2x+1} = \left(\frac{1}{2}\right)^x$

Solution: (c) Rewrite the problem so that the base is the same on both sides.

$2^{2x+1} = \left(\frac{1}{2}\right)^x$ $2^{2x+1} = \left(2^{-1}\right)^x = 2^{-x}$ Now you can equate the exponents since both sides have base 2. $2x + 1 = -x$ $x = -\frac{1}{3}$

6.6 E. Solve for x: $5^x = 3^{1-2x}$

Solution: (c) Take the natural log of both sides.

$5^x = 3^{1-2x}$ $\ln(5^x) = \ln(3^{1-2x})$ $x \ln 5 = (1 - 2x) \ln 3$

$x \ln 5 + 2x \ln 3 = \ln 3$ $(\ln 5 + 2 \ln 3)x = \ln 3$ $x = \frac{\ln 3}{\ln 5 + 2 \ln 3}$