

SOLVED PROBLEMS: SECTION 1.1

17. Show that in any ring R the commutative law for addition is redundant, in the sense that it follows from the other axioms for a ring.
18. An element $a \in R$ is said to be *nilpotent* if $a^k = 0$ for some positive integer k . Determine the nilpotent elements of \mathbf{Z}_n .
19. Show that for any ring R the diagonal elements $\{(a, a) \mid a \in R\}$ form a subring of the direct sum $R \oplus R$.
20. Let R be a ring, and let X be any subset of R . Show that the following subset is a subring of R : $C(X) = \{r \in R \mid rx = xr \ \forall x \in X\}$.
21. Show that if $\{R_\alpha\}_{\alpha \in I}$ is any collection of subrings of the ring S , then the intersection $\bigcap_{\alpha \in I} R_\alpha$ is a subring of S .
22. Let α be an algebraic integer that is a root of $p(x) = x^n + b_{n-1}x^{n-1} + \cdots + b_0$ in $\mathbf{Z}[x]$. Show that $\mathbf{Z}[\alpha] = \{c_{n-1}\alpha^{n-1} + c_{n-2}\alpha^{n-2} + \cdots + c_1\alpha + c_0 \mid c_i \in \mathbf{Z}\}$ is a subring of \mathbf{C} .
23. Let R be a ring, let S be a subring of R , and let I be an ideal of R . Show that the following subset is a subring of R : $S + I = \{s + a \mid s \in S, a \in I\}$.
24. Let a, b be elements of the ring R . Show that $1 - ab$ is a unit if and only if $1 - ba$ is a unit. If this is the case, find $(1 - ab)^{-1}$.
25. Prove that the ring R is commutative if $x^2 - x \in Z(R)$, for all $x \in R$.