13. Let $P$ be a prime ideal of the commutative ring $R$. Prove that if $P$ is a prime ideal of $R$, then $A \cap B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$, for all ideals $A, B$ of $R$. Give an example to show that the converse is false.

14. Show that in the polynomial ring $\mathbb{Z}[x]$, the ideal $(n, x)$ generated by $n \in \mathbb{Z}$ and $x$ is a prime ideal if and only if $n$ is a prime number.

15. Let $R$ be a Boolean ring (see Exercise 1.1.11 in the text) and let $P$ be a prime ideal of $R$. Prove that $P$ is maximal, and that $R/P \cong \mathbb{Z}_2$.

16. Let $R$ be a commutative ring. Then $R$ is called a local ring if it has a proper ideal $P$ such that $P \supset I$, for all proper ideals $I$ of $R$. Prove that the following conditions are equivalent for $R$.

   (1) $R$ is a local ring;
   (2) the set of nonunits of $R$ forms an ideal;
   (3) there exists a maximal ideal $P$ of $R$ such that $1 + x$ is a unit, for all $x \in P$.

17. Prove that any nonzero homomorphic image of a local ring is again a local ring.

18. Show that the ring $R$ defined in Exercise 1.2.9 of the text is a local ring.