

## CHAPTER 3: STRUCTURE OF NONCOMMUTATIVE RINGS

## Review Problems

1. Let  $R$  be a left Artinian ring in which  $I^2 = (0)$  implies  $I = (0)$ , for all ideals  $I$  of  $R$ . Prove that  $R$  is a semisimple Artinian ring.
2. Let  $R$  be a semisimple Artinian ring. Prove that if  $ab = 0$  implies  $a = 0$  or  $b = 0$ , for all  $a, b \in R$ , then  $R$  is a division ring.
3. Let  $R$  be a ring, let  ${}_R M$  be a Noetherian  $R$ -module, and let  $J$  be the Jacobson radical of  $R$ . Prove that either  $J^n M = (0)$  for some positive integer  $n$ , or else the descending chain  $M \supset JM \supset J^2 M \supset \cdots$  is an infinite chain, showing that  $M$  is not Artinian.
4. Show that if  $I$  and  $J$  are nilpotent ideals of the ring  $R$ , then the ideal  $I + J$  is also nilpotent.
5. Let  $F$  be a field, let  $V$  be an infinite dimensional vector space over  $F$ , and let  $R$  be the idealization of  $V$ , as defined in Exercise 2.1.12 of the text. Compute the Jacobson radical of  $R$ .
6. Prove that any homomorphic image of a semisimple Artinian ring is again semisimple Artinian.