

Find the derivatives of the following functions:

$$e^x \sin(e^x - e^{-x}) \quad \ln(3x^2(x^2 - 2)) \quad x \tan(e^x) \quad \frac{\operatorname{arcsec}(x^2)}{x} \quad e^{\sqrt{x^2+1}}$$

$$(x-1)\ln(x-1) \quad \cot(\tan(x)) \quad x^2 \sin^4(x) \quad \ln(\arcsin x) \quad \ln \sqrt{\frac{3x-1}{3}}$$

Find the limits:

$$\lim_{x \rightarrow 0} \frac{\cos(3x)-1}{2x} \quad \lim_{x \rightarrow \infty} x^{-2} \ln(x^x) \quad \lim_{x \rightarrow 0} (1-2x)^{1/x} \quad \lim_{n \rightarrow \infty} n \ln\left(1 + \frac{4}{n}\right)$$

$$\lim_{x \rightarrow 0^+} x \ln x \quad \lim_{n \rightarrow \infty} \arctan(n\pi) + e^{-n\pi} \quad \lim_{x \rightarrow 2} \frac{d}{dx} \int_1^x \frac{1}{t+1} dt \quad \lim_{n \rightarrow \infty} \frac{n^3}{(n+1)!}$$

If a population of bacteria quadruples every 100 hours, and you start with 1,000 bacteria, when approximately will you have ten million of them?

20% of a radioactive isotope decays in 10 years; what half-life does that substance have?

It takes 12 lbs. of force to stretch an ideal spring from its natural length by 4 feet. How much work is required to stretch it from its natural length by 3 feet?

If work required to compress a spring from its natural length by 0.5 m is 10 Nm, what force has to be applied to hold it in that state?

Find the area of the closed region between the curves  $y = \frac{1}{1+x^2}$  and  $y = \frac{x^2}{1+x^2}$  (sketch the graphs!)

The right half ( $x \geq 0$ ) of the region in the previous problem is rotated about the  $y$  axis. Find the volume of the resulting solid.

Find the average value of  $\sqrt{4-x^2}$  between  $x = -2$  and  $x = 2$ .

Find the following integrals:

$$\int \frac{\ln(x^2)}{x} dx \quad \int_0^2 e^x \sin(e^x) dx \quad \int (x+1)(3x-1)^{2/3} dx \quad \int \operatorname{arcsec}(3x) dx$$

$$\int \frac{1}{-6+4x-x^2} dx \quad \int \frac{4}{(x^2+9)^2} dx \quad \int \sin(2x) \cos(3x) dx \quad \int_1^\infty 4xe^{-x} dx$$

$$\int_3^4 \frac{x}{\sqrt{x^2-9}} dx \quad \int \tan^3 x dx \quad \int \cos^2 x \cot x dx \quad \int \frac{3x^3 - x^2 + 8x - 4}{x^4 + 4x^2} dx$$

Determine convergence of each of the series:

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n + \ln n} \quad \sum_{n=1}^{\infty} \frac{2^{n-1}}{\ln \sqrt{n}} \quad \sum_{n=0}^{\infty} (-1)^n \frac{27^{n/2}}{n6^n} \quad \sum_{n=1}^{\infty} \frac{1}{n \ln n^2}$$

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n(\sqrt{n}+1)} \quad \sum_{n=0}^{\infty} \frac{(-e)^n}{n(e+1)^n} \quad \sum_{n=2}^{\infty} \frac{1}{n^2-n} \quad \sum_{n=0}^{\infty} \frac{n^2 \arctan(-n)}{(2n+1)(n+2)}$$

Find the interval of convergence of the series  $\sum_{n=1}^{\infty} (-2)^{-n} \frac{(2x-1)^n}{n \ln n}$ .

Write down the first 5 terms of the Taylor series for  $f(x) = \sin(2x)$  around  $a = 0$ . Use this to find the first 5 terms of the Taylor series for  $g(x) = 2 \cos(2x)$ .