

4. Let y be the solution to the problem:

$$y' = 2x + 3y - 1; \quad y(0) = 2$$

(A) Use the Euler method to estimate $y(.2)$ and taking $h = .1$

(B) Use the improved Euler method to estimate $y(.2)$ and taking $h = .1$

1. Find the Laplace Transforms for:

a. $f(t) = t \cos(3t)$.

b. $f(t) = \frac{\sin(2t)}{t}$.

2. Solve using the Laplace transform

$$y'' + 2y' + 2y = 1 \quad y(0) = 3, \quad y'(0) = 0.$$

3. (20 Pts.) A body of mass $m = 2$ is on a frictionless horizontal plane and can move only back and forth. This body is attached both to a spring with constant $k = 40$ and a dashpot with damping constant $c = 16$. The mass is set in motion with initial position $x_0 = 5$ and initial velocity $v_0 = 4$.

- a. Find the differential equation satisfied by the position function $x(t)$.
- b. Find the position function $x(t)$.

4. (20 Pts.) Find the Inverse Laplace Transforms for:

a. $F(s) = \frac{s - 11}{(s - 1)(s^2 + 4)}$

b. $F(s) = \frac{1}{s^2(s - 3)}$ (using Convolution).

5. (20 Pts) Find the steady periodic solution in the form $x_{sp}(t) = C \cos(\omega t - \alpha)$ with $C > 0$.

$$2x'' + x' + 100x = -2020 \sin(10t)$$