1. The following table gives the steps necessary to complete a task. For each task it lists the time (in hours) to complete the task and the immediately preceeding tasks.

<table>
<thead>
<tr>
<th>Task</th>
<th>Time</th>
<th>Preceeding steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>none</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>none</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>A, B</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
<td>C</td>
</tr>
<tr>
<td>E</td>
<td>7</td>
<td>C</td>
</tr>
<tr>
<td>F</td>
<td>3</td>
<td>E</td>
</tr>
<tr>
<td>G</td>
<td>4</td>
<td>D, E</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>G, F</td>
</tr>
</tbody>
</table>

(a) Draw the PERT diagram for this problem.
(b) What is the critical path?
(c) What would the critical path be if the time for task E was reduced to 4 hours?

2. Counting problems (From Section 1.2 - forgot to list that).

(a) From the standard 26 letter alphabet, how many different 3 letter initial sets could be made? (For instance my initials are JBS)
(b) How many of the above use no letter more than once?
(c) If \( S = \{1, 2, 3, 4\} \), then how many permutations of three elements of the set are possible?

3. Let \( U = \{1, 2, 3, 4, 5, 6, 7, 8\} \) be the universal set, and consider the following subsets: \( A = \{2, 4, 5, 7\} \), \( B = \{1, 2, 3, 4\} \), \( C = \{1, 4, 6, 7, 8\} \). Find the following subsets.

(a) \( \bar{A} \)
(b) \( A \cup B \)
(c) \( (A \cap B) \cup C \)
(d) \( (A \cup B) \cap C \)

4. Draw a Venn diagram for the set \( (\bar{A} \cap B) \cup (A \cap \bar{B}) \), where \( A \) and \( B \) are subsets of a universal set \( U \). This set is sometimes called the symmetric difference of the sets \( A \) and \( B \).

5. When can you be sure that \( A \cap B = A \)?

6. If \( X \) has 5 elements and \( Y \) has 7 elements, then how many elements does \( X \times Y \) have?
7. Define the following for a set $X$:

(a) A relation $R$ on $X$,

(b) the following properties that a relation $R$ may or may not have.
   i. the relation $R$ is reflexive.
   ii. the relation $R$ is symmetric.
   iii. the relation $R$ is antisymmetric.
   iv. the relation $R$ is transitive.

(c) Define: $R$ is an equivalence relation. (Which of the properties does it have?)

(d) Define: $R$ is a partial order. (Which of the properties does it have?)

8. Define the relation $\sim$ on $Z$, the set of integers, by $x \sim y$ if and only if 7 divides $x - y$.

(a) First, give a useful definition of “7 divides the integer $n$”.

(b) Now prove that $\sim$ is an equivalence relation on $Z$.

9. Let $S = \{2, 3, 4, 6, 8, 9, 12, 16, 18\}$, and define the relation $\preceq$ on $S$ by $x \preceq y$ if and only if $y = x \cdot 2^k$ for some integer $k = 0, 1, 2, 3, \ldots$.

(a) Prove that $\preceq$ is a partial order on the positive integers. (we are considering it for a subset of the integers $S$, but prove it in general.)

(b) Draw a Hasse diagram for the resulting partially ordered set.

(c) identify, specifically, any maximum, maximal, minimum or minimal elements.

10. Functions.

(a) Define $f : X \rightarrow Y$ is a function.

(b) the function $f$ is one-to-one.

(c) $f$ is an onto function.

11. Let $X = \{1, 2, 3\}$ and $Y = \{a, b, c, d\}$

(a) draw a diagram (or use ordered pair notation) for each of the following.
   i. A function from $X$ to $Y$ (any one will do)
   ii. a relation that is not a function from $X$ to $Y$.
   iii. a one-to one function from $X$ to $Y$.

(b) Is it possible for there to be an onto function from $X$ to $Y$? How about $Y$ to $X$?