This is practice for exam 1 in Math 206.
This is long compared to what you can expect.

1. The following table gives the steps necessary to complete a task. For each task it lists the time (in hours) to complete the task and the immediately preceeding tasks.

Task	Time	Preceeding steps
A	2	none
B	4	none
C	3	A, B
D	6	C
E	7	C
F	3	E
G	4	D, E
H	1	G, F

   (a) Draw the PERT diagram for this problem.

   (b) What is the critical path?

   (c) What would the critical path be if the time for task E was reduced to 4 hours?

2. Counting problems

   (a) From the standard 26 letter alphabet, how many different 3 letter ordered initial sets could be made? (For instance my initials are JBS)

   (b) How many of the above use no letter more than once?

   (c) If $S = \{1, 2, 3, 4\}$, then how many permutaions of three elements of the set are possible?

   (d) How many combinations of 3 elements from the set S in the previous problem may be formed?

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ be the universal set, and consider the following subsets: $A = \{2, 4, 5, 7\}$, $B = \{1, 2, 3, 4\}$, $C = \{1, 4, 6, 7, 8\}$. Find the following subsets.

   (a) $\bar{A}$

   (b) $A \cup B$

   (c) $(A \cap B) \cup C$

   (d) $(A \cup B) \cap C$

4. When can you be sure that $A \cap B = A$?

5. If X has 5 elements and Y has 7 elements, then how many elements does $X \times Y$ have?

6. Define the following for a set X:

   (a) A relation $\mathcal{R}$ on X,

   (b) the following properties that a relation $\mathcal{R}$ may or may not have.

      i. the relation $\mathcal{R}$ is reflexive.

      ii. the relation $\mathcal{R}$ is symmetric.

      iii. the relation $\mathcal{R}$ is antisymmetric.

      iv. the relation $\mathcal{R}$ is transitive.

   (c) Define: $\mathcal{R}$ is an eqivalence relation. (Which of the properties does it have?)

   (d) Define: $\mathcal{R}$ is a partial order. (Which of the properties does it have?)

7. Define the relation $\sim$ on Z, the set of integers, by $x \sim y$ if and only if 7 divides $x - y$.

   (a) First, give a useful definition of "7 divides the integer n".

   (b) Now prove that $\sim$ is an equivalence relation on Z.

8. Let $S = \{2, 3, 4, 6, 8, 9, 12, 16, 18\}$, and define the relation $\preceq$ on S by $x \preceq y$ ifa and only if $y = x \cdot 2^k$ for some integer $k = 0, 1, 2, 3, \ldots$.
   (a) Prove that $\preceq$ is a partial order on the positive integers. (we are condiering it for a subset of the integers S, but prove it in general.)
   (b) Draw a Hasse diagram for the resulting partially ordered set.
   (c) identify, specifically, any maximum, maximal, mininimum or minimal elements.
9. Functions.
   (a) Define $f : X \to Y$ is a function.
   (b) the function f is one-to-one.
   (c) f is an onto function.
10. Let $X = \{1, 2, 3\}$ and $Y = \{a, b, c, d\}$
   (a) draw a diagram (or use ordered pair notation) for each of the following.
      i. A function from X to Y (any one will do)
      ii. a relation that is not a function from X to Y.
      iii. a one-to one function from X to Y.
   (b) Is it possible for there to be an onto function from X to Y? How about Y to X?
11. Find the negations of each of the following.
   (a) All students work hard and some professors are entertaining.
   (b) If you are industrious, the you will get ahead.
   (c) If every nontrivial divisor of n is even, then n is even. (nontrivial means not n or 1)
   (d) Every integer is dvisible by a prime number.
   (e) Some polynomials cannot be factored.
12. Write $\sum_{k=1}^{4}(2k^2 - 1)$ as a sum of four integers.
13. Prove by mathemtatical induction that $\displaystyle\sum_{k=1}^{n}(2k - 1) = n^2$
14. What is the coefficient of u^7v^3 in the expansion of $(u - 3v)^{10}$?

EXTRA PROBLEMS:

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 0\}$ be the universal set, and consider The following subsets: $A = \{2, 4, 5, 0\}$, $B = \{1, 2, 3, 4\}$, $C = \{1, 2, 6, 7, 8\}$. Find the following subsets.
   (a) Draw a careful Venn diagram for the set U and its subsets A, B, and C. Be VERY careful in putting the numbers in the correcr regions and subregions!

(b) List the elements (using set notation) for each of the following (with a little work you can read them from the diagram.)

i. The complement of $A \cup B \cup C$

ii. $A \cup B$

iii. $A \cap \bar{B}$

iv. $(A \cup B) \cap C$

2. If $X = \{0, a\}$ and $Y = \{7, M\}$, then list ALL elements of $X \times Y$.

3. Let $X = \{f(x) = ax^2 + bx + c\}$, the set of all quadratic polynomials.
For X define the relation $f(x) \sim g(x)$ if and only if $f(0) = g(0)$. Prove, carefully, that $\sim$ is an equivalence relation on X. (Name the properties that you prove it has.)

4. Let X be the set of integers $\{2, 3, 6, 9, 18\}$. The relation $a \preceq b$ if and only if $a|b$ (a divides b). This is a partial order.

(a) Give a useful, functional definition of *a divides b*.

(b) Draw a Hasse diagram for this partially ordered set.

(c) identify, specifically, any maximum, maximal, mininimum or minimal elements.

5. Prove: The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 17x - 33$ is both one-to-one and onto.

6. Let X and Y be finite sets with m and n elements, respectively.

(a) If there is a one-to-one function $f : X \to Y$, then how do m and n compare in size?

(b) When is there a one-to-one and onto function from X to Y?

(c) If EVERY function from X to Y is onto, when what can you say about n?

7. Explain why an n element set has 2^n different possible subsets.

8. suppose that we have two functions $f : X \to Y$ and $g : Y \to Z$ where $X = \{1, 2, 3\}$, $Y = \{a, b, c, d, f\}$ and $Z = \{P, U\}$. If $f(1) = b$, $f(2) = c$, $f(3) = f$, and $g(a) = g(b) = g(c) = g(d) = P$ and $g(f) = U$, then what is $g \circ f(1)$? $g(f(3))$?

Is $f \circ g$ defined? Why or why not?