

1. Define a is congruent to b modulo n in two different, but equivalent ways.
2. Consider the following problems on congruence.
 - (a) Which of the following are congruent to 3 modulo 7: 19, 24, 31 and/or -11?
 - (b) For $a = 5$, $b = 13$, find r and s , $0 \leq r, s < 8$, such that $a + b \equiv r \pmod{8}$ and $a \cdot b \equiv s \pmod{8}$.
 - (c) Give an example of $a \not\equiv 0 \pmod{30}$ and $b \not\equiv 0 \pmod{30}$, but $a \cdot b \equiv 0 \pmod{30}$.
3. State the division algorithm, and apply it to -31 divided by 7.
4. Let $a = 310$ and $b = 54$.
 - (a) Use the Euclidean Algorithm to find the greatest common divisor (gcd) of a and b : $\gcd(a, b)$.
 - (b) Find x and y such that $ax + by = \gcd(a, b)$
5. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 0 \\ 3 & 4 & 0 \\ 1 & 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 4 & 0 \\ 3 & 4 & 0 & 3 \\ 1 & 1 & 2 & -1 \end{bmatrix}$$
 - (a) Compute each of the following, or explain why it is undefined. $A+B$, $A+C$.
 - (b) Compute each of the following, or explain why it is undefined. AB , BC , CB .
6. Find two 2×2 matrices, A and B such that $AB \neq BA$.
7. Using your basic knowledge and the rules of formal logic determine which of the following are true.
 - (a) October 23 is Sunday or there is a test scheduled for Thursday October 25 in MATH 206.
 - (b) If today is February 29th, then everyone will pass their MATH 206 test.
 - (c) World Famous makes the best pizza in DeKalb.
8. Find the negations of each of the following statements.
 - (a) Today is Thursday and tomorrow is Monday.
 - (b) Every good boy deserves favor.

- (c) If Dave passes his French class then he will graduate.
- (d) All children are well behaved or some dogs bite.
9. For the logical statement *if it rains, then John Street will flood* find (and label) the inverse, the converse and the contrapositive. Which of these three is logically equivalent (is true at exactly the same times as) the original statement?
10. Prove by mathematical induction that

$$1 + 7 + 7^2 + \cdots + 7^n = \frac{7^{n+1} - 1}{7 - 1}$$

11. Give an example of a graph with four vertices labeled A, B, C, D where A has degree 3, B and C have degree 2 and D has degree 1. Now write down the adjacency matrix for this graph (with vertices corresponding to rows and columns in alphabetic order).
12. For the adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Draw the graph \mathcal{G} represented. Label your vertices 1, 2, 3, 4, and 5, corresponding to row and/or column numbers.