Little Results from the

Axioms for Integers

Lemma: The additive identity is unique.

To see this, try adding two putative additive identities together. What do you get?

Lemma: For any $a \in \mathbb{Z}$, the additive inverse of a is unique.

To see this, add a to two putative inverses, getting 0. Now add the first inverse to each sum. What do you get?

Once we know there is only one "negative a", we can give it a name: $-a$.

Lemma: For any $a \in \mathbb{Z}$, $a \times 0 = 0$.

To see this, look at $(0 + 0) \times a$.

Lemma: For all $a \in \mathbb{Z}$, $-1 \times a = -a$.

To see this, see what happens when you multiply $(1 + -1)$ by a using what we already know.

Lemma: If $a > 0$, then $-a < 0$. If $a < 0$, then $-a > 0$.

Lemma: If $a \times b = 0$, then either $a = 0$ or $b = 0$.

To see this, show that $a \times b = 0$ implies that $a \times b = a \times (-b) = -(a \times b)$.

Lemma: If $a \times b = a \times c$ and $a \neq 0$, then $b = c$.

Lemma: The multiplicative identity is unique.

Lemma: The multiplicative identity is the least positive integer.

First, there is a least positive integer by our last axiom. Call it a for the time being. Certainly $a \leq 1$. Show that $a^2 \leq a$. Why must this be an equality? What does that imply?