Math 230 **Quiz 2 Solutions** **Spring 2003**

1. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

$$\int_0^4 x dy = 2\int_0^b x dy \Longrightarrow \int_0^4 y^{1/2} dy = 2\int_0^b y^{1/2} dy$$

$$\Longrightarrow \frac{2}{3}\left[y^{3/2}\right]_0^4 = 2\frac{2}{3}\left[y^{3/2}\right]_0^b$$

$$\Longrightarrow \frac{2}{3}(8-0) = \frac{4}{3}(b^{3/2}-0) \Longrightarrow b^{3/2} = \frac{6}{12}8 = \frac{48}{12} = 4 \Longrightarrow b = 4^{2/3}$$

Or: $\int_0^2 (4-x^2)dx = 2\int_0^{\sqrt{b}} (4-x^2)dx$

$$\Longrightarrow 4x - \frac{1}{3}x^3|_0^2 = 2(4x - \frac{1}{3}x^3)|_0^{\sqrt{b}}$$

$$\Longrightarrow 8 - \frac{8}{3} = 2(4\sqrt{b} - \frac{1}{3}b^{3/2}) \Longrightarrow \frac{16}{3} = 8b^{1/2} - \frac{2}{3}b^{3/2}$$

2. Find the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 0$, $x = 2$, $x = 4$ about the line $x = 1$.

For $0 \le y < 2$, a cross-section is an annulus with inner radius $(2-1)$ and outer radius $(4-1)$, the area of which is

$$A_1(y) = \pi(4-1)^2 - \pi(2-1)^2$$

For $2 \le y \le 4$ a cross-section is an annulus with inner radius $y-1$ and outer radius $(4-1)$, the area of which is

$$A_2(y) = \pi(4-1)^2 - \pi(y-1)^2$$

$$\Longrightarrow V = \pi\int_0^2 [(4-1)^2 - (2-1)^2]dy + \pi\int_2^4 [(4-1)^2 - (y-1)^2]dy$$

$$= \pi[8y]_0^2 + \pi\int_2^4 (8+2y-y^2)dy = 16\pi + \pi\left[8y + y^2 - \frac{1}{3}y^3\right]_2^4$$

$$16\pi + \pi\left[\left(32+16-\frac{64}{3}\right) - \left(16+4-\frac{8}{3}\right)\right] = \frac{76\pi}{3}$$

Or: $\int_2^4 2\pi(x-1)x dx = \int_2^4 2\pi(x^2-x)dx = (2\pi\frac{1}{3}x^3 - \frac{1}{2}x^2)|_2^4 = \left[\frac{64}{3} - 8 - \left(\frac{8}{3} - 2\right)\right]2\pi$

$$= \left(\frac{56}{3} - 6\right)2\pi = \frac{38}{3}2\pi = \frac{76\pi}{3}$$

3. A force of 10 lb. is required to hold a spring stretched 4 in. beyond its natural length. How much work is done in stretching it from its natural length to 6 in. beyond its natural length?

$10 = f(x) = kx = \frac{1}{3}k$ (4 inches = 1/3 foot), so $k = 30$lb/ft and $f(x) = 30x$. Now 6 inches = 1/2 foot, so $W = \int_0^{1/2} 30x dx = [15x^2]_0^{1/2} = \frac{15}{4}$ ft-lb.