Math 230 **Quiz 3** **Spring 2003**

Name: ____________________________________

You must SHOW YOUR WORK in order to receive credit.

1. (5 pts) Find the inverse function of $f(x) = \dfrac{5x-1}{3+2x}$

   Solution: $y = \dfrac{5x-1}{3+2x} \Longrightarrow y(3+2x) = 5x-1 \Longrightarrow 3y+2xy = 5x-1 \Longrightarrow -5x+2xy = -3y-1 \Longrightarrow x(-5+2y) = -3y-1 \Longrightarrow x = \dfrac{-3y-1}{2y-5} \Longrightarrow f^{-1}(x) = \dfrac{-3y-1}{2y-5}$

2. (5 pts) Use the inverse function theorem (no other method will receive credit) to find $g'(8)$ for $f(x) = x^3$ where $g = f^{-1}$

   Solution: $x^3 = 8 \Longrightarrow x = 8^{1/3} \Longrightarrow x = 2 \Longrightarrow g(a) = g(8) = 2 \Longrightarrow f(x) = x^3 \Longrightarrow f'(x) = 3x^2 \Longrightarrow g'(8) = \dfrac{1}{f'((g(8))} = \dfrac{1}{f'(2)} = 1/12$

3. (5 pts) Find $\dfrac{d}{dx}\left(\dfrac{\sin^2 x \tan^4 x}{(x^2+1)^2}\right)$

   Solution: $y = \dfrac{\sin^2 x \tan^4 x}{(x^2+1)^2}$

$$\Longrightarrow \ln y = \ln\left(\frac{\sin^2 x \tan^4 x}{(x^2+1)^2}\right)$$

$$= \ln(\sin^2 x \tan^4 x) - \ln(x^2+1)^2$$

$$= \ln \sin^2 x + \tan^4 x - \ln(x^2+1)^2$$

$$= 2\ln \sin x + 4\ln \tan x - 2\ln(x^2+1)$$

$$\Longrightarrow \frac{y'}{y} = 2\frac{\cos x}{\sin x} + 4\frac{\sec^2 x}{\tan x} - 2\frac{2x}{x^2+1}$$

$$\Longrightarrow y' = \left[2\cot x + 4\frac{\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right]\left[\frac{\sin^2 x \tan^4 x}{(x^2+1)^2}\right]$$

4. (5 pts) Evaluate $\int e^x\sqrt{1+e^x}dx$

   Solution: $u = 1+e^x \Longrightarrow du = e^x dx \Longrightarrow \int \frac{2}{3}u^{3/2} + c$

$$\Longrightarrow \frac{2}{3}(1+e^x)^{3/2} + c$$