

Name: _____

You must SHOW YOUR WORK in order to receive credit.

1. $\int \frac{\log_{10} x}{x} dx$

Solution: $= \int \frac{\ln x / \ln 10}{x} dx = \frac{1}{\ln 10} \int \frac{\ln x}{x} dx$

$$\implies u = \ln x \implies du = \frac{1}{x} dx \implies \frac{1}{\ln 10} \int u du = \frac{1}{\ln 10} \frac{1}{2} u^2 + c = \frac{1}{2 \ln 10} (\ln x)^2 + c$$

2. $\frac{d}{dx} (\sin x)^x$

Solution: $\implies \ln y = x \ln(\sin x) \implies \frac{y'}{y} = \ln(\sin x) + \frac{x \cos x}{\sin x}$

$$\implies y' = (\sin x)^x [\ln(\sin x) + x \cot x]$$

3. A substance has a half-life of 140 days. If the original mass was 200 mg, find a formula for the mass that remains after t days.

Solution: $y(140) = 200e^{140k} = 100 \implies e^{140k} = \frac{1}{2} \implies 140k = \ln \frac{1}{2}$

$$\implies k = -\frac{\ln 2}{140} \implies y(t) = 200e^{-(\ln 2)t/140} = 200(2^{-t/140})$$

4. For the same substance, find the mass after 100 days.

Solution: $y(100) = 200(2^{-100/140})$ which is approximately 121.9 mg