Math 230-1
Review: Key Definitions, Theorems, and Formulas
Spring 2003

1. **Definition 1** *A function $f(x)$ is continuous at a if:*

   (a) $f(x)$ *is defined at* $x = a$

   (b) $\lim_{n\to\infty} f(x)$ *exists and is finite*

   (c) $\lim_{n\to\infty} = f(a)$

2. **Theorem 1 (Composition Theorem)** *If* $\lim_{x\to a} g(x) = b$ *and* $f(x)$ *is continuous at* b*, then* $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x)) = f(b)$.

3. **Theorem 2 (Intermediate Value Theorem)** *Let* $f(x)$ *be continous on the closed interval* $[a, b]$ *and let* N *be any number between* $f(a)$ *and* $f(b)$*. Then there exists* $c \in (a, b)$ *such that* $f(c) = N$.

4. **Rule 1 (The Product Rule)** *The derivative of the product of two functions equals the first times the derivative of the second plus the second times the derivative of the first.*

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

5. **Rule 2 (The Quotient Rule)** *The derivative of the quotient of two functions equals the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator all divided by the denominator squared.*

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

6. **Rule 3 (The Chain Rule)** *The derivative of a composition of two functions equals the derivative of the outside function evaluated at the inside function times the derivative of the inside function.*

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

7. for any rational number r, $\frac{d}{dx}(x^r) = rx^{r-1}$

8. **Theorem 3 (Mean Value Theorem)** *If* $f(x)$ *is continuous on* $[a, b]$ *and differentiable on* (a, b)*, then there is a* $c \in (a, b)$ *such that* $f(b) - f(a) = f(c)(b - a)$.

9. Trigonometric Identities:

(a) Fundamental Identities:

i. $\csc x = \dfrac{1}{\sin x}$

ii. $\sec x = \dfrac{1}{\cos x}$

iii. $\tan x = \dfrac{\sin x}{\cos x}$

iv. $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

v. $\sin^2 x + \cos^2 x = 1$

vi. $\sec^2 x = 1 + \tan^2 x$

vii. $\cot^2 x + 1 = \csc^2 x$

(b) Negative Angle Identities

i. $\sin(-x) = -\sin x$

ii. $\cos(-x) = \cos x$

iii. $\tan(-x) = -\tan x$

iv. $\csc(-x) = -\csc x$

v. $\sec(-x) = \sec x$

vi. $\cot(-x) = -\cot x$

(c) Cofunction Identities: If $A + B = \dfrac{\pi}{2}$

i. $\sin A = \cos B$

ii. $\sec A = \csc B$

iii. $\tan A = \cot B$

(d) Addition and Subtraction Identities

i. $\sin(x + y) = \sin x \cos y + \cos x \sin y$

ii. $\cos(x + y) = \cos x \cos y - \sin x \sin y$

iii. $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

iv. $\sin(x - y) = \sin x \cos y - \cos x \sin y$

v. $\cos(x - y) = \cos x \cos y + \sin x \sin y$

vi. $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

(e) Double Angle Identities

i. $\sin 2x = 2 \sin x \cos x$

ii. $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$

iii. $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

(f) Half-Angle Identities

i. $\sin \dfrac{x}{2} = +/- \sqrt{\dfrac{1 - \cos x}{2}}$

ii. $\cos\frac{x}{2} = +/- \sqrt{\frac{1+\cos x}{x}}$

iii. $\tan\frac{x}{2} = +/- \sqrt{\frac{1-\cos x}{1+\cos x}}$

(g) Product Identities

i. $\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$

ii. $\cos x \sin y = \frac{1}{2}(\sin(x+y) - \sin(x-y))$

iii. $\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$

iv. $\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$

(h) Sum Identities

i. $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

ii. $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

iii. $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

iv. $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

10. Derivatives of Trig Functions:

(a) $\frac{d}{dx}\sin x = \cos x$

(b) $\frac{d}{dx}\cos x = -\sin x$

(c) $\frac{d}{dx}\tan x = \sec^2 x$

(d) $\frac{d}{dx}\cot x = -\csc^2 x$

(e) $\frac{d}{dx}\sec x = \sec x \tan x$

(f) $\frac{d}{dx}\csc x = -\csc x \cot x$