Math 230 **EXAM 1** **Spring 2003**

Name: _____

Make sure your exam has all 4 pages. You must SHOW YOUR WORK in order to receive credit.

1. (15 pts) Find the number b such that $y = b$ doubles the region bounded by the curves $y = x^3$, $y = 8$, and $x = 0$.

2. (15 pts) Consider the region bounded by $y = \sqrt{x}$, $x = 4$, $x = 16$ rotated about $x = 1$.

   (a) Set up but DO NOT evaluate the integral for the disk/washer method.

   (b) Set up but DO NOT evaluate the integral for the cylindrical shell method.

   (c) Find the volume using either (a) or (b).

3. (8 pts) Let $f(x) = x^5 + 3x^3 + 2x + 9$. Find $(f^{-1})'(15)$.

4. (8 pts) Use logarithmic differentiation to find the derivative of the function $y = (\tan x)^x$ defined on the interval $(0, \frac{\pi}{2})$.

5. (10 pts) Differentiate the following functions:

   (a) $f(x) = \ln(x + \sqrt{x^2 - 1})$

   (b) $f(x) = e^x \ln x$

6. (8 pts) Find the equation of the line tangent to the curve $f(x) = x^3 e^x$ at the point $(1, e)$.

7. (20 pts) Integrate the following functions:

   (a) $\int (e^x + 1)dx$

   (b) $\int e^x \sin(e^x)dx$

   (c) $\int \frac{x^2 + 1}{x^3 + 3x + 1}dx$

   (d) $\int xe^{x^2}dx$

8. (16 pts) Let $P(t)$ be the number of bacteria in a certain culture after t hours. Suppose that initially there are 2,000 bacteria and the culture grows at a rate proportional to its size, doubling every 3 hours.

   (a) Find the growth constant.

   (b) After how many hours will there be 20,000 bacteria?

   (c) How many bacteria are there after 12 hours?