

Lecture notes: 6.3: Volumes by Cylindrical Shells

Some problems

1. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$

Solution:

- (a) draw picture
(b) determine limits of integration

$$2x^2 - x^3 = 0 \implies x^2(2 - x) = 0 \implies x = 0, x = 2$$

- (c) set up integral

$$V = \int_0^2 (2\pi x)(2x^2 - x^3) dx = 2\pi \int_0^2 (2x^3 - x^4) dx$$

- (d) evaluate integral

$$= 2\pi \left[\frac{1}{2}x^4 - \frac{1}{5}x^5 \right]_0^2 = 2\pi \left(8 - \frac{32}{5} \right) = \frac{16\pi}{5}$$

2. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$

Solution:

- (a) draw picture
(b) determine limits of integration

$$x = x^2 \implies x^2 - x = 0 \implies x(x - 1) = 0 \implies x = 0, x = 1$$

- (c) set up integral

$$V = \int_0^1 (2\pi x)(x - x^2) dx = 2\pi \int_0^1 (x^2 - x^3) dx$$

- (d) evaluate the integral

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = \frac{\pi}{6}$$

3. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$

Solution:

- (a) draw picture
- (b) determine the limits of integration

$$x - x^2 = 0 \implies x(1 - x) = 0 \implies x = 0, x = 1$$

- (c) set up the integral

$$V = \int_0^1 2\pi(2 - x)(x - x^2)dx = 2\pi \int_0^1 (x^3 - 3x^2 + 2x)dx$$

- (d) evaluate the integral

$$= 2\pi \left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1 = \frac{\pi}{2}$$

4. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 0$, $x = 1$, $x = 2$, about $x = 4$

Solution:

- (a) draw picture
- (b) determine the limits of integration
- (c) set up integral

$$V = \int_1^2 2\pi(4 - x)x^2 dx = 2\pi \int_1^2 (4x^2 - x^3) dx$$

- (d) evaluate integral

$$= 2\pi \left[\frac{4}{3}x^3 - \frac{1}{4}x^4 \right]_1^2 = 2\pi \left[\left(\frac{32}{3} - 4 \right) - \left(\frac{4}{3} - \frac{1}{4} \right) \right] = \frac{67\pi}{6}$$

5. The integral $\int_0^9 2\pi y^{3/2} dy$ represents a solid - describe it.

Solution: The solid is obtained by rotating the region bounded by the curve $x = \sqrt{y}$ and the lines $y = 9$ and $x = 0$ about the x-axis

6. $y = 5$, $y = x^2 - 5x + 9$ about $x = -1$

solution:

(a) draw picture

(b) determine limits of integration

$$x^2 - 5x + 9 = 5 \implies x^2 - 5x + 4 = 0 \implies (x-4)(x-1) = 0 \implies x = 4, x = 1$$

(c) set up integral

$$\begin{aligned} V &= \int_1^4 2\pi[x - (-1)][5 - (x^2 - 5x + 9)]dx = 2\pi \int_1^4 (x+1)(-x^2 + 5x - 4)dx \\ &= 2\pi \int_1^4 (-x^3 + 4x^2 + x - 4)dx \end{aligned}$$

(d) evaluate integral

$$= 2\pi \left[\left(-64 + \frac{256}{3} + 8 - 16 \right) - \left(-\frac{1}{4} + \frac{4}{3} + \frac{1}{2} - 4 \right) \right] = 2\pi \left(\frac{63}{4} \right) = \frac{63\pi}{2}$$

Homework: 10, 20, 36, 40