Concepts necessary for the homework: **DISCLAIMER** This is a summary, it is not intended to be comprehensive or complete. It is meant to help you organize the material so that you can effectively study for the exam.

§13.1

1. describe and sketch surfaces in three dimensions
2. distance formula for three dimensions
   distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$
$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
3. equation of a sphere with center $C(h, k, l)$ and radius r
$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

§13.2

1. relationship between vectors and points
2. given two points, find a vector
   initial point $P(p_1, p_2, p_3)$ and terminal point $Q(q_1, q_2, q_3)$ has component form
$$< q_1 - p_1, q_2 - p_2, q_3 - p_3 >$$
3. for $\mathbf{v} =< v_1, v_2, v_3 >$, $\mathbf{u} =< u_1, u_2, u_3 >$, $c \in \mathbb{R}$ find:
   (a) sum of two vectors
   $$\mathbf{u} + \mathbf{v} =< u_1 + v_1, u_2 + v_2, u_3 + v_3 >$$
   (b) scalar multiples of vectors
   $$c\mathbf{u} =< cu_1, cu_2, cu_3 >$$
   (c) norm of a vector
   $$|\mathbf{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$
   (d) normalized vector
   $$\frac{\mathbf{u}}{|\mathbf{u}|}$$

§13.3

given $\mathbf{v} =< v_1, v_2, v_3 >$, $\mathbf{u} < u_1, u_2, u_3 >$, θ the angle between $\mathbf{u}$ and $\mathbf{v}$

1. compute the dot product of two vectors given:
   (a) two vectors
   $$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$$
   (b) magnitude of two vectors and the angle between them
   $$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
2. find the angle between two vectors
3. determine if two vectors are perpendicular, parallel, or neither
   $\mathbf{u}$ and $\mathbf{v}$ are perpendicular if and only if their dot product is 0
   $\mathbf{u}$ and $\mathbf{v}$ are parallel if one is a scalar multiple of the other, or if $\cos\theta = \pm 1$

4. compute the direction angles and direction cosines of a vector

$$\cos\alpha = \frac{v_1}{|\mathbf{v}|}$$

$$\cos\beta = \frac{v_2}{|\mathbf{v}|}$$

$$\cos\gamma = \frac{v_3}{|\mathbf{v}|}$$

where α is the angle between $\mathbf{v}$ and $\mathbf{i}$, β is the angle between $\mathbf{v}$ and $\mathbf{j}$, γ is the angle between $\mathbf{v}$ and $\mathbf{k}$

5. compute scalar and vector projections for two vectors

Scalar projection of $\mathbf{v}$ onto $\mathbf{u}$: $$comp_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|}$$

Vector projection of $\mathbf{v}$ onto $\mathbf{u}$: $$proj_{\mathbf{u}}\mathbf{v} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|}\right)\frac{\mathbf{u}}{|\mathbf{u}|}$$

§13.4

given $\mathbf{v} =< v_1, v_2, v_3 >$, $\mathbf{u} < u_1, u_2, u_3 >$, $\mathbf{w} =< w_1, w_2, w_3 >$, θ the angle between $\mathbf{u}$ and $\mathbf{v}$

1. compute the cross product of two vectors given two vectors

$$\mathbf{u}\times\mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

i.e. need to be able to set up and compute the determinant

2. find the magnitude of the cross product given the magnitude of two vectors and the angle between them

$$|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}||\mathbf{v}|\sin\theta$$

3. determine the direction of the cross product

use the right hand rule

4. find a vector orthogonal to a plane defined by three given points P, Q, R

construct the vectors $\vec{PQ}$ and $\vec{PR}$ and then take their cross product

5. find the volume V of a parallelepiped determined by three vectors $\mathbf{u}$, $\mathbf{v}$ $\mathbf{w}$

$$V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|$$

6. use scalar triple product to determine if points are coplanar

construct three vectors from the four points and three vectors are coplanar if

$$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = 0$$

§13.5

1. find vector equation, parametric equations and symmetric equations for a line given:

   (a) a vector and a point
   a line L parallel to $\mathbf{v} =< a, b, c >$ and passing through the point $P(x_1, y_1, z_1)$

$$\begin{cases} x = x_1 + at \\ y = y_1 + bt \\ z = z_1 + ct \end{cases}$$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$< x_1 + at, y_1 + bt, z_1 + ct >$$

(b) two points
construct the component form of the vector from the two given points and use the equations above

(c) two planes (i.e. find the line of intersection of two planes)
plane one: $a_1x + b_1y + c_1z = d_1$
plane two: $a_2x + b_2y + c_2z = d_2$
first find a point that lies on both planes $P(x_1, y_1, z_1)$. The normal vector for plane one is $\mathbf{u} =< a_1, b_1, c_1 >$ and the normal vector for plane two is $\mathbf{v} =< a_2, b_2, c_2 >$ and $\mathbf{w} = \mathbf{u} \times \mathbf{v}$ is parallel to L. Then we can use P and $\mathbf{w}$ with the equations above to construct the equations for the line.

2. determine if two lines are perpendicular, parallel, or skew

   to be parallel, corresponding vectors must be multiples of each other

   to be perpendicular, dot product of corresponding vectors must be 0

   it they are not parallel or perpindicular, they are skew

3. find the equation of the plane given

   (a) point and normal vector
   the plane containing the point $P(x_1, y_1, z_1)$ and having normal vector
   $\mathbf{n} =< a, b, c >$ has equation

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

   (b) three points P, Q, R
   construct $\mathbf{u} = \vec{PQ}$ and $\mathbf{v} = \vec{PR}$. Then $\mathbf{u} \times \mathbf{v}$ is orthogonal to the plane and so can be used as the normal vector together with P as above.

   (c) point and line of intersection of two planes

4. determine if planes are perpendicular, parallel, neither, or identical

   plane one: $a_1x + b_1y + c_1z = d_1$

   plane two: $a_2x + b_2y + c_2z = d_2$

   The normal vector for plane one is $\mathbf{u} =< a_1, b_1, c_1 >$ and the normal vector for plane two is $\mathbf{v} =< a_2, b_2, c_2 >$. So θ, the angle between the normal vectors, is given by $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$. Then answer based on the relationship of the vectors.

5. find the angle between two planes

   the angle between two planes is the same as the angle between their normal vectors

6. find the distance between a point and a plane

   the distance D from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is given by

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$