Stuff for studying for the final

Although all the sections are not covered here, most of the problems require information from other sections. You can use this in conjunction with previous summary sheets and review sheets to prepare for the final exam.

§11.3

1. Plot polar coordinates
2. convert Cartesian ↔ polar (both points and equations)

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\tan\theta = \frac{y}{x}$$
$$r^2 = x^2 + y^2$$

Problems: 29-35

§11.4

1. find area bounded by polar curves

$$A = \int_a^b \frac{1}{2} r^2 d\theta$$

2. sketch polar curves and use to find area inside of one curve and outside another, or inside both curves, or find all points of intersection of two curves
3. find arc length for polar equations

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Problems: 9-14, 45-48

§13.2

1. relationship between vectors and points
2. given two points, find a vector
   initial point $P(p_1, p_2, p_3)$ and terminal point $Q(q_1, q_2, q_3)$ has component form

$$< q_1 - p_1, q_2 - p_2, q_3 - p_3 >$$

3. for $\mathbf{v} =< v_1, v_2, v_3 >$, $\mathbf{u} =< u_1, u_2, u_3 >$, $c \in \mathbb{R}$ find:
   (a) sum of two vectors
   $$\mathbf{u} + \mathbf{v} =< u_1 + v_1, u_2 + v_2, u_3 + v_3 >$$
   (b) scalar multiples of vectors
   $$c\mathbf{u} =< cu_1, cu_2, cu_3 >$$
   (c) norm of a vector
   $$|\mathbf{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$
   (d) normalized vector
   $$\frac{\mathbf{u}}{|\mathbf{u}|}$$

Problems: 7-12

§13.3

given $\mathbf{v} =< v_1, v_2, v_3 >$, $\mathbf{u} < u_1, u_2, u_3 >$, θ the angle between $\mathbf{u}$ and $\mathbf{v}$

1. compute the dot product of two vectors given:
   (a) two vectors
   $$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$
   (b) magnitude of two vectors and the angle between them
   $$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
2. find the angle between two vectors
3. determine if two vectors are perpendicular, parallel, or neither

   $\mathbf{u}$ and $\mathbf{v}$ are perpendicular if and only if their dot product is 0

   $\mathbf{u}$ and $\mathbf{v}$ are parallel if one is a scalar multiple of the other, or if $\cos\theta = \pm 1$

Problems: 15-20

§13.4

given $\mathbf{v} =< v_1, v_2, v_3 >$, $\mathbf{u} < u_1, u_2, u_3 >$, $\mathbf{w} =< w_1, w_2, w_3 >$, θ the angle between $\mathbf{u}$ and $\mathbf{v}$

1. compute the cross product of two vectors given two vectors
   $$\mathbf{u} \times \mathbf{v} = (u_2 v_3 - u_3 v_2)\mathbf{i} - (u_1 v_3 - u_3 v_1)\mathbf{j} + (u_1 v_2 - u_2 v_1)\mathbf{k}$$
   i.e. need to be able to set up and compute the determinant
2. find the magnitude of the cross product given the magnitude of two vectors and the angle between them
   $$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta$$
3. determine the direction of the cross product

   use the right hand rule
4. find a vector orthogonal to a plane defined by three given points P, Q, R

   construct the vectors $\vec{PQ}$ and $\vec{PR}$ and then take their cross product

Problems: 3-7

§13.5

1. find vector equation, parametric equations and symmetric equations for a line given:
   (a) a vector and a point

   a line L parallel to $\mathbf{v} =< a, b, c >$ and passing through the point $P(x_1, y_1, z_1)$
   $$\begin{cases} x = x_1 + at \\ y = y_1 + bt \\ z = z_1 + ct \end{cases}$$
   $$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$
   $$< x_1 + at, y_1 + bt, z_1 + ct >$$
   (b) two points

   construct the component form of the vector from the two given points and use the equations above

(c) two planes (i.e. find the line of intersection of two planes)
plane one: $a_1x + b_1y + c_1z = d_1$
plane two: $a_2x + b_2y + c_2z = d_2$
first find a point that lies on both planes $P(x_1, y_1, z_1)$. The normal vector for plane one is $\mathbf{u} =< a_1, b_1, c_1 >$ and the normal vector for plane two is
$\mathbf{v} =< a_2, b_2, c_2 >$ and $\mathbf{w} = \mathbf{u} \times \mathbf{v}$ is parallel to L. Then we can use P and $\mathbf{w}$ with the equations above to construct the equations for the line.

2. determine if two lines are perpendicular, parallel, or skew

   to be parallel, corresponding vectors must be multiples of each other

   to be perpendicular, dot product of corresponding vectors must be 0 and the lines must intersect

   it they are not parallel or perpindicular, they are skew

3. find the equation of the plane given

   (a) point and normal vector
   the plane containing the point $P(x_1, y_1, z_1)$ and having normal vector
   $\mathbf{n} =< a, b, c >$ has equation

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

   (b) three points P, Q, R
   construct $\mathbf{u} = \vec{PQ}$ and $\mathbf{v} = \vec{PR}$. Then $\mathbf{u} \times \mathbf{v}$ is orthogonal to the plane and so can be used as the normal vector together with P as above.

   (c) point and line of intersection of two planes

4. determine if planes are perpendicular, parallel, neither, or identical

   plane one: $a_1x + b_1y + c_1z = d_1$

   plane two: $a_2x + b_2y + c_2z = d_2$

   The normal vector for plane one is $\mathbf{u} =< a_1, b_1, c_1 >$ and the normal vector for plane two is
   $\mathbf{v} =< a_2, b_2, c_2 >$. So θ, the angle between the normal vectors, is given by $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$. Then answer based on the relationship of the vectors.

5. find the angle between two planes

   the angle between two planes is the same as the angle between their normal vectors

6. find the distance between a point and a plane

   the distance D from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is given by

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Problems: 23-30

§14.2

1. Draw $\mathbf{r}(t)$ for a specific value of t, differences between vectors, derivative vectors and tangent vectors.

   See figure 1 on page 893 and figure 2 on page 894.

2. Sketch the plane curve with a given vector equation, find $\mathbf{r}'(t)$

   for $\mathbf{r}(t) =< f(t), g(t), h(t) >$, $\mathbf{r}'(t) =< f'(t), g'(t), h'(t) >$

3. Find the unit vector $\mathbf{T}(t)$ at a point with a given parameter value of t.

   First, find $\mathbf{r}'(t)$. Next, plug the value of t specified into $\mathbf{r}'(t)$. Then divide by the norm of the resulting vector.

4. Find parametric equations for the tangent line to the curve with given parametric equations at a specified point.

   Need a vector the line is parallel to and a point it passes through. To get the vector, it is $\mathbf{r}'(t)$ evaluated at the t value that results in the given point. The point is the one given to you. Construct the parametric equations from the formulas given in §13.5.

5. Integrate a vector equation.

   $$\int_a^b \mathbf{r}(t)dt = \left(\int_a^b f(t)dt\right)\mathbf{i} + \left(\int_a^b g(t)dt\right)\mathbf{j} + \left(\int_a^b h(t)dt\right)\mathbf{k}$$

Problems: 23-26

§14.4

1. Find the velocity, acceleration, and speed of a particle with a given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t

   velocity= $\mathbf{v}(t) = \mathbf{r}'(t)$

   acceleration= $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$

   speed= $|\mathbf{v}(t)| = |\mathbf{r}'(t)|$

   see figures 1,2, and 3 from the section for how to draw the vectors.

2. Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.

   Given $\mathbf{a}(t)$, $\mathbf{v}(\alpha) = c$, $\mathbf{r}(\alpha) = d$

   First integrate $\mathbf{a}(t)$ to get $\mathbf{v}(t) + \mathbf{C}$. Then plug in $t = \alpha$ and solve for $\mathbf{C}$. Plug $\mathbf{C}$ into $\mathbf{v}(t) + \mathbf{C}$, and integrate to get $\mathbf{r}(t) + \mathbf{D}$. Then plug in $t = \alpha$ and solve for $\mathbf{D}$.

3. Find max/min speed of a position vector.

   speed= $|\mathbf{v}(t)| = |\mathbf{r}'(t)|$, so first take the derivative of $\mathbf{r}(t)$. Then find $|\mathbf{t}|$. Take the derivative of $|\mathbf{t}|$. Find the critical points and test for max and min.

Problems: 9-13

§15.2

1. Evaluate limits for functions of more than one variable

   Remember to plug in first. The only problem situation is when you have 0 in the denominator.

   (a) To show a limit does not exist, you must find two paths which yield different limits. You should be able to evaluate along the paths:
      i. $y = 0$
      ii. $x = 0$
      iii. $y = mx$
      iv. $y = x^2$
      v. $x = y^2$
   (b) To show a limit exists and find its value, you should be able to employ the same techniques as used in single variable analysis:
      i. factor and cancel
      ii. multiply by 1 (usually involves the conjugate)
      iii. use the squeeze theorem
   (c) When all the simple techniques fail, use the $\delta - \epsilon$ definition.

2. Find the set on which a function is continuous.

   A function of more than one variable is continuous where the limit exists and is equal to the funcion value at that point. For composite functions $g(f(x,y))$, if f is continuous at (x_0, y_0) and g is a single variable function continuous at $g(f(x_0, y_0))$.

Problems: 7-16

§15.4

1. Find an equation of the tangent plane to a given surface at a given point.

   Suppose f has continuous partial derivatives. An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Problems: 1-6

§15.5

1. Use the chain rule to find derivatives or partial derivatives.

   Given $z = f(x, y)$ a differentiable function of x and y, where $x = x(t)$ and $y = y(t)$ are both differentiable functions of t

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

   Given $z = f(x, y)$ a differentiable function of x and y, where $x = x(s, t)$ and $y = y(s, t)$ are both differentiable functions of s and t

$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$
$$\frac{\partial z}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

   Given $u = f(x_1, x_2, \dots, x_n)$ a differentiable function of x_i for $1 \le i \le n$, where $x_i = x(t_1, t_2, \dots, t_m)$ are differentiable functions of t_j

$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}$$

   for each $i = 1, 2, \dots, m$

2. Be able to do implicit differentiation for multivariable functions.

   Given a function $F(x, y)$ where y is implicitly defined as a function of x

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

   Given a function $F(x, y, z)$ where z is implicitly defined as a function of x and y

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$
$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Problems: 21-22

§15.6

1. Find the directional derivative of f at the given point in the direction indicated by the angle θ

   Given $f(x, y)$, $P(a, b)$, and θ, the directional derivative is given by

$$D_u f(a, b) = f_x(a, b) \cos \theta + f_y(a, b) \sin \theta$$

2. Given $f(x, y, z)$, $P(a, b, c)$, $\mathbf{v} =< v_1, v_2, v_3 >$

   (a) Find the gradient of f

   (b) Evaluate the gradient at the point P

   (c) Find the rate of change of f at P in the direction of the vector $\mathbf{v}$

$$\nabla f =< f_x, f_y, f_z >$$

$$\nabla f(a, b, c) =< f_x(a, b, c), f_y(a, b, c), f_z(a, b, c) >$$

$$D_u f(a, b, c) = \nabla f(a, b, c) \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|} \right)$$

3. Find the maximum and minimum rates of change of f at a given point and the direction in which it occurs.

$$D_u f = \nabla f \cdot \mathbf{u} = |\nabla f||\mathbf{u}| \cos \theta = |\nabla f| \cos \theta$$

   $\Rightarrow$ the maximum value of $D_u f$ is $|\nabla f|$ and it occurs when $\theta = 0$, which is when $\mathbf{u}$ has the same direction as ∇f

4. Find equations of the tangent plane and the normal line to a given surface at a specified point.

   The tangent plane to the level surface $F(x_0, y_0, z_0) = k$ at the point $P(x_0, y_0, z_0)$ is the plane that passes through P and has normal vector $\nabla F(x_0, y_0, z_0)$

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

   The normal line to S at P is the line passing through P and perpendicular to the tangent plane. It's symmetric equations are

$$\frac{x - x_0}{F_x(x_0, y_0, z_0)} = \frac{y - y_0}{F_y(x_0, y_0, z_0)} = \frac{z - z_0}{F_z(x_0, y_0, z_0)}$$

Problems: 21-26

§15.7

1. The connection between level curves of a function and the location of maxima, minima, and saddle points of that function.

   A function of two variables has a local maximum at (a, b) if $f(x, y) \leq f(a, b)$ when (x, y) is near (a, b). The number $f(a, b)$ is called a local maximum value. If $f(x, y) \geq f(a, b)$ when (x, y) is near (a, b), then $f(a, b)$ is a local minimum value.

   If f has a local maximum or minimum at (a, b) and the first-order partial derivative of f exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

   A point (a, b) is called a critical point of f if $f_x(a, b) = 0$ and $f_y(a, b) = 0$, or if one of these partial derivatives does not exist.

   Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

   (a) $D > 0$ and $f_{xx}(a, b) > 0 \Rightarrow f(a, b)$ is a local minimum.

(b) $D > 0$ and $f_{xx}(a, b) < 0 \Rightarrow f(a, b)$ is a local maximum.

(c) $D < 0$, then $f(a, b)$ is not a local maximum or minimum (in which case the point (a, b) is called a saddle point of f and the graph of f crosses its tangent plane at (a, b).

2. Find local maximum and minimum values and saddle points of a given function using the above information.

Problems: 5-11

§15.8

1. Use Lagrange multipliers to find the maximum and minimum values of a given function subject to given constraints

   To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ [assuming these extreme values exist and $\nabla g \neq 0$ on the surface $g(x, y, z) = k$:

   (a) Find all values of x, y, z, and λ such that

   $$\nabla f(x, y, z) = \lambda g(x, y, z)$$

   and

   $$g(x, y, z) = k$$

   i.e. solve the system of equations: $\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ g(x, y, z) = k \end{cases}$

   (b) Evaluate f at all the points (x, y, z) that result from the first step. The largest of these values is the maximum value of f; the smallest is the minimum values of f.

Problems: 4-9

§16.3

1. Set up and evaluate double integrals with non-constant limits of integration (over general regions) (see discussion after §16.4)

2. Find the volume of a given solid

   The volume of the solid bounded below by the section R of the xy-plane and above by the surface $z = f(x, y)$ is given by $\int\int_R f(x, y)dA$. If you are finding the volume of the region between two surfaces, you have to figure out which is on top to determine which integral is subtracted from which.

3. Sketch regions of integration given Cartesian equations

4. Change the order of integration

§16.4

1. Sketch regions of integration given polar equations

2. Change Cartesian double integrals to polar double integrals

   (a) $x = r\cos\theta$

   (b) $y = r\sin\theta$

   (c) $r^2 = x^2 + y^2$

   (d) $\tan\theta = y/x$

   (e) $dydx$ or $dxdy$ become $rdrd\theta$

3. Evaluate polar double integrals

Process for setting up double integrals:

1. Look at $z = f(x,y)$. Does it have to be integrated in a specific order? (i.e. $f(x,y) = e^{x^2}$ cannot be integrated with respect to x first). Is it more easily represented in the polar coordinate system? (i.e. $f(x,y) = e^{x^2+y^2}$ has the less complex representation of e^{r^2}).
2. Sketch the region R, which is determined by the single variable functions you are given. Determine if one direction of integration is easier than the other (i.e. do you have to do one integral in this direction but two integrals in that direction?). Determine if R is easily represented in polar coordinates (i.e. are you integrating over a circle?).
3. Based on (1) and (2), choose a coordinate system and order of integration. The outer integral will have constant limits of integration and the inner integral will have functions of the outer integral variable (which can be constant).
   (a) if you are using the rectangular system
      i. The outer limits are determined by the largest and smallest value the variable takes on in R.
      ii. Draw a line through R perpendicular to the axis of the outer variable. Where this line enters R is the lower boundary and where it leaves R is the upper boundary for the inner integral.
   (b) if you are using the polar system
      i. draw a ray through R that begins at the origin. Where it enters R is the lower bound and where it leaves R is the upper bound for the inner integral
      ii. the θ limits are determined by the angles where the region starts (lower) and finishes (upper)

§16.5

Given $\rho(x,y)$ as the density function for a lamina that occupies R

1. Total charge/mass
   $m = \int\int_R \rho(x,y)dA$
2. Center of mass
   $\overline{x} = \dfrac{M_y}{m}$, $\overline{y} = \dfrac{M_x}{m}$ where
   $M_y = \int\int_R x\rho(x,y)dA$ and $M_x = \int\int_R y\rho(x,y)dA$
3. Moments of inertia
   $I_x = \int\int_R y^2\rho(x,y)dA$
   $I_y = \int\int_R x^2\rho(x,y)dA$
   $I_0 = \int\int_R (x^2+y^2)\rho(x,y)dA = I_x + I_y$

§16.6

1. Find surface area
   $SA = \int\int_R \sqrt{1+f_x^2+f_y^2}dA$

Problems: 1-6

§16.7

1. Set up and evaluate triple integrals in the rectangular coordinate system

2. Sketch solid based on a given triple integral
3. Change limits of integration
4. Find mass and center of mass given a density function ρ

Problems: 35-38

§16.8

1. Determine limits for a triple integral over a region
2. Conversion between rectangular, cylindrical, and spherical coordinate systems
   Cylindrical to rectangular:
   (a) $x = r\cos\theta$
   (b) $y = r\sin\theta$
   (c) $z = z$

   Spherical to rectangular:
   (a) $x = \rho\sin\phi\cos\theta$
   (b) $y = \rho\sin\phi\sin\theta$
   (c) $z = \rho\cos\phi$

   dV in triple integrals
   (a) rectangular: $dV = dx\ dy\ dz$
   (b) cylindrical: $dV = r\ dz\ dr\ d\theta$
   (c) spherical: $dV = \rho^2\ \sin\phi\ d\rho\ d\phi\ d\theta$
3. Find volume of solids

Evaluating a triple integral:
Look at $f(x, y, z)$. Which coordinate system is it easiest to represent the function? Look at Q, the region you are integrating over, which system is it easiest to represent in? (i.e. are you integrating over a sphere? a cone? a cylinder?)

triple integrals: given $f(x, y, x)$ integrated over a three dimensional region Q. The projection of Q onto the xy-plane R can be visualized by thinking of the sun shining directly over Q and casting a shadow onto the xy-plane. Evaluation of triple integrals is done by iterated integrals and is the same process as iterated integrals for double integrals.

1. rectangular
   (a) sketch the region Q.
   (b) decide whether it is vertically simple or horizontally simple. This determines the order of integration. Suppose it is vertically simple. Then you integrate with respect to z for the innermost integral, and its projection R would be onto the xy-plane. If, however, it is horizontally simple, then the projection R will be either onto the xz-plane or the yz-plane.
   (c) Now refer to double integrals.
2. cylindrical
   (a) sketch the region Q along with its projection R onto the xy-plane.
   (b) find the z limits of integration. Draw a line M through a typical point (r, θ) of R parallel to the z-axis. As z increases, M enters Q at the lower limit of integration and leaves Q at the upper limit. These limits are functions of r and θ.

(c) find the r and θ limits the same way you find these limits for double integrals in polar coordinates.

3. spherical

   (a) sketch the region Q along with its projection R onto the xy-plane.

   (b) find the ρ limits of integration. Draw a ray M from the origin through Q making an angle ϕ with the positive z-axis. Also draw the projection of M on the xy-plane (L). The ray L makes an angle θ with the positive x-axis. As ρ increases, M enters Q at the lower bound and leaves at the upper bound. These limits are functions of ϕ and θ.

   (c) find the *phi* limits of integration. For any given θ, the angle ϕ that M makes with the z-axis runs from the lower limit to the upper limit. These are functions of θ.

   (d) find the θ limits of integration. The ray L sweeps over R as θ runs from the lower bound to the upper bound.

Problems: 7-10, 17-20

§16.9

1. The Jacobian of the transformation T given by $x = g(u, v)$ and $y = h(u, v)$ is

$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

2. Use transformations to evaluate integrals. Suppose that T is a C^1 transformation whose Jacobian is nonzero and that maps a region S in the uv-plane onto a region R in the xy-plane. Suppose that f is continuous on R. Suppose also that T is one-to-one, except perhaps on the boundary of S. Then

$$\int\int_R f(x, y)dA = \int\int_S f(x(u, v), y(u, v))|J|dudv$$

Problems: 11-13, 19,20

§17.2

1. If f is defined on a smooth curve C given by $x = x(t)$, $y = y(t)$, $z = z(t)$, $a \leq t \leq b$, then the line integral of f along C is given by

$$\int_C f(x, y, z)dS = \int_a^b f(x(t), y(t), z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

2. The line integral of f along C with respect to x is given by

$$\int_C f(x, y, z)dx = \int_a^b f(x(t), y(t), z(t))x'(t)dt$$

3. The line integral of f along C with respect to y is given by

$$\int_C f(x, y)dy = \int_a^b f(x(t), y(t), z(t))y'(t)dt$$

4. The line integral of f along C with respect to z is given by

$$\int_C f(x, y)dz = \int_a^b f(x(t), y(t), z(t))z'(t)dt$$

5. Let $\mathbf{F} =< P, Q, R >$ be a continuous vector field defined on a smooth curve C given by a vector function $\mathbf{r}(t)$, $a \leq t \leq b$. Then the line integral of $\mathbf{F}$ along C is

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C Pdx + Qdy + Rdz$$

Problems: 7-11

§17.3

1. Let C be a smooth curve joining the points A and B in the plane or in space and given by $\mathbf{r}(t)$. Let F be a differentiable function with a continuous gradient vector $\mathbf{F} = \nabla f$ on a domain containing C. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A)$$

2. Component test for conservative fields (two-dimensional): Let $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$ be a field on a connected and simply connected domain whose component functions have continuous first partials. Then $\mathbf{F}$ is conservative if and only if

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

3. Component test for conservative fields (three-dimensional): Let $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ be a field on a connected and simply connected domain whose component functions have continuous first partials. Then $\mathbf{F}$ is conservative if and only if

$$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}, \qquad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}, \qquad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

Problems: 17.3/3-6, 17.5/13-18

§17.4

1. Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C. If P and Q have continuous partial derivatives on an open region that contains D, then

$$\int_C Pdx + Qdy = \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Problems: 7-12