Exam I Reveiw Sheet

While working these problems, note the length of time it takes to do them. You will only have 50 minutes for the exam, and so should do the problems in the order of the speed with which you can complete them to maximize the number of points you receive.

The following basic trig identies will be the only formulas provided for you. They will be provided in the following forms:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \qquad \sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y \qquad \cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \qquad \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$
$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\sin x \cos x = \frac{1}{2}\sin(2x) \qquad \sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$
$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)] \qquad \cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

1. For the following parametric equations:
   (a) Sketch a curve given parametric equations (be sure to indicate direction)
   (b) Eliminate the parameter to find a Cartesian equation of the curve
      i. $x = 3t - 5$, $y = 2t + 1$
      ii. $x = t^2 - 2$, $y = 5 - 2t$
      iii. $x = 1 - t$, $y = \sqrt{t}$
      iv. $x = 2 - t^2$, $y = 1 + 3t$
      v. $x = t^2$, $y = t^3$
2. For the following parametric equations:
   (a) Find an equation of the tangent line to a curve at a given point
   (b) determine concavity of the curve
   (c) find points on graph where tangents are vertical or horizontal
      i. $x = e^{\sqrt{t}}$, $y = t - \ln(t^2)$, $t = 1$
      ii. $x = \cos\theta + \sin 2\theta$, $y = \sin\theta + \cos 2\theta$, $\theta = 0$
      iii. $x = e^t$, $y = (t-1)^2$, $(1, 1)$
      iv. $x = 2\sin 2t$, $y = 2\sin t$, $(\sqrt{3}, 1)$
      v. $x = t + \ln t$, $y = t - \ln t$, $(1, 1)$
      vi. $x = 2\cos\theta$, $y = \sin 2\theta$, $t = \pi/4$
3. For the following parametric curves, find the arc length and area under the curve on the given intervals where it is possible. Where it is not possible, just set up the equation.
   (a) $x = t - t^2$, $y = \frac{4}{3}t^{3/2}$, $1 \le t \le 2$
   (b) $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \le t \le 1$
   (c) $x = e^t \cos t$, $y = e^t \sin t$, $0 \le t \le \pi$
   (d) $x = t + \cos t$, $y = t - \sin t$, $0 \le t \le 2\pi$
   (e) $x = \cos t + \ln\left(\tan\frac{t}{2}\right)$, $y = \sin t$, $\frac{\pi}{4} \le t \le \frac{3\pi}{4}$

4. Convert the following from polar to Cartesian or Cartesian to polar depending on which you are given and classify the curve (i.e. a circle centered at (h, k) of radius r).

   (a) $r = 3\sin\theta$
   (b) $r = 2\sin\theta + 2\cos\theta$
   (c) $r = \tan\theta\sec\theta$
   (d) $x^2 - y^2 = 1$
   (e) $x^2 + y^2 = 9$
   (f) $x = -y^2$

5. Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

   (a) $r = 2\sin\theta$, $\theta = \pi/6$
   (b) $r = 1/\theta$, $\theta = \pi$
   (c) $r = 1 + \cos\theta$, $\theta = \pi/4$
   (d) $r = 2 - \sin\theta$, $\theta = \pi/3$
   (e) $r = \ln\theta$, $\theta = e$

6. Sketch the following polar curves and find the designated area.

   (a) The area that lies inside of $r = 4\sin\theta$ and outside of $r = 2$
   (b) The area that lies inside of $r = 1 - \sin\theta$ and above $\theta = 0$
   (c) The area that lies inside of both $r = \sin\theta$ and $r = \cos\theta$
   (d) The area that lies inside of both $r = 3 + 2\sin\theta$ and $r = 2$
   (e) The area enclosed by one loop of $r = 3\cos 5\theta$
   (f) The area enclosed by $r = 2 + \cos 2\theta$ and above the x-axis.

7. for the following vectors and scalars, find:

   (a) $|\mathbf{u}|$
   (b) normalize $\mathbf{u}$
   (c) $|\mathbf{v}|$
   (d) normalize $\mathbf{v}$
   (e) $a\mathbf{u} + b\mathbf{v}$
   (f) $\mathbf{u} \cdot \mathbf{v}$
   (g) $\mathbf{u} \times \mathbf{v}$
   (h) $proj_{\mathbf{u}}\mathbf{v}$
   (i) determine if $\mathbf{v}$ and $\mathbf{u}$ are orthogonal, perpendicular or neither and justify your answer

      i. $\mathbf{v} =< 1, 2, 3 >$, $\mathbf{u} =< -1, 3, 0 >$, $a = 2$, $b = -4$
      ii. $\mathbf{v} =< 2, 1, 0 >$, $\mathbf{u} =< 4, 7, 2 >$, $a = 3$, $b = 2$
      iii. $\mathbf{v} =< 4, 2, 6 >$, $\mathbf{u} =< 2, 1, 3 >$, $a = -3$, $b = 2$
      iv. $\mathbf{v} =< 6, -3, 2 >$, $\mathbf{u} =< 2, 1, -2 >$, $a = 7$, $b = -3$
      v. $\mathbf{v} =< -3, 9, 6 >$, $\mathbf{u} =< 4, -12, -8 >$, $a = -2$, $b = 4$
      vi. $\mathbf{v} =< -3, 0, 9 >$, $\mathbf{u} =< -1, 0, -3 >$, $a = 3$, $b = -6$

8. Given the following, find $\mathbf{u} \cdot \mathbf{v}$ and $|\mathbf{u} \times \mathbf{v}|$

   (a) $|\mathbf{u}| = 5$, $|\mathbf{v}| = 6$, $\theta = \pi/2$
   (b) $|\mathbf{u}| = 2$, $|\mathbf{v}| = 12$, $\theta = \pi/3$
   (c) $|\mathbf{u}| = 3$, $|\mathbf{v}| = 1$, $\theta = \pi/4$
   (d) $|\mathbf{u}| = 2$, $|\mathbf{v}| = 16$, $\theta = \pi/6$

(e) $|\mathbf{u}| = 14$, $|\mathbf{v}| = 13$, $\theta = \pi/4$

9. find vector equation, parametric equations and symmetric equations for the line specified.

   (a) parallel to $\mathbf{v}$ and passing through the point P

      i. $\mathbf{v} =< -2, 3, 1 >$, $P(6, -7, 2)$
      ii. $\mathbf{v} =< -3, 5, 1 >$, $P(4, 2, 1)$
      iii. $\mathbf{v} =< 0, -5, 16 >$, $P(-1, -4, 2)$
      iv. $\mathbf{v} =< -2, 0, -11 >$, $P(2, -1, 0)$
      v. $\mathbf{v} =< -6, 13, 0 >$, $P(3, 1, 4)$

   (b) passes through the points P and Q

      i. $P(1, 3, 2)$, $Q(-1, 0, 2)$
      ii. $P(-2, 0, 3)$, $Q(-1, 2, -1)$
      iii. $P(10, 3, -2)$, $Q(1, 1, -1)$
      iv. $P(2, -3, 4)$, $Q(-1, 2, -3)$
      v. $P(-1, -3, 0)$, $Q(0, -3, -1)$

10. Determine if the above lines (pair a i with b i and so on) are parallel, perpendicular, or skew.

11. Find the equation to the specified plane

   (a) through the point P with normal vector $\mathbf{v}$

      i. $\mathbf{v} =< -2, -3, 1 >$, $P(-6, -7, 2)$
      ii. $\mathbf{v} =< 2, 3, 7 >$, $P(-3, 1, -2)$
      iii. $\mathbf{v} =< 1, -5, 16 >$, $P(1, -4, 2)$
      iv. $\mathbf{v} =< 2, 0, -11 >$, $P(2, 1, 0)$
      v. $\mathbf{v} =< 6, 13, 0 >$, $P(3, 1, -4)$

   (b) through the point P and parallel to the given plane

      i. $P(0, 0, 0)$, $2x - y + 3z = 1$
      ii. $P(1, 1, 0)$, $-x + 2y + = 4$
      iii. $P(-1, 2, -1)$, $2y - 3z = 7$
      iv. $P(1, -3, 1)$, $2x + 3y - 3z = 12$

   (c) that contains the points P, Q, and R

      i. $P(1, 1, 2)$, $Q(2, -1, 3)$, $R(1, 2, 6)$
      ii. $P(-1, 0, 2)$, $Q(0, -1, 3)$, $R(1, 2, 0)$
      iii. $P(-1, 2, 1)$, $Q(-2, -1, 0)$, $R(-1, 1, 3)$
      iv. $P(-3, 1, 0)$, $Q(2, 0, 3)$, $R(1, 2, 6)$