MATH 434/534

Theoretical Assignment 1
Due on September 8th, 2011

Chapter 2 Page 25-28.

No. 2.0
Answer ”‘True”’ or ”‘False”’ to the following. Give reasons for your answers.

(a) The eigenvalues of an upper triangular matrix T are its diagonal entries.
(b) The eigenvalues of a real symmetric matrix are real.
(c) A matrix is nonsingular if and only if all its eigenvalues are nonzero.
(d) The eigenvalues of an orthogonal matrix are all equal to 1.
(e) An orthogonal matrix is not necessarily invertible.
(f) A real symmetric or a complex Hermitian matrix can be always transformed into a diagonal matrix by similarity transformation.
(g) Two similar matrices have the same eigenvalues.
(h) If two matrices have the same eigenvalues, they must be similar.
(i) The product of two upper (lower) triangular matrices does not need to be an upper (lower) triangular matrix.
(j) $\|I\| = 1$ for any norm.
(k) The length of a vector is preserved by an orthogonal multiplication (multiplication of a vector or a matrix by an orthogonal matrix).
(l) If $\|A\| < 1$, then $I - A$ is nonsingular.
(m) If $\|A\|_2 = 1$, then A must be orthogonal.
(n) The product of two orthogonal (unitary) matrices is an orthogonal (unitary) matrix.

No. 2.10
Prove that for a subordinate matrix norm $\|\cdot\|$, $|\lambda| \leq \|A\|$ for every eigenvalues of λ of A.
No. 2.22
If x and y are two vectors, then prove that
(a) $\left|x^T y\right| \leq \|x\|_2 \|y\|_2$ (Cauchy-Schwarz inequality)

No. 2.23
Let x and y be two orthogonal vectors. Then prove that $\|x+y\|_2^2 = \|x\|_2^2 + \|y\|_2^2$

No. 2.28
Prove that (i) $\|I\|_2 = 1$, and (ii) $\|I\|_F = \sqrt{n}$.

No. 2.29
Prove that if Q and P are orthogonal matrices, then (a) $\|QAP\|_F = \|A\|_F$, and (b)$\|QAP\|_2 = \|A\|_2$

No. 2.33
Prove that (i) $\left\|A^T\right\|_2 = \|A\|_2$, and (ii) $\left\|A^T A\right\|_2 = \|A\|_2^2$

No. 2.34
Let $A = (a_1, \ldots, a_N)$, where a_j is the j^{th} column of A. Then prove that $\|A\|_F^2 = \sum_{i=1}^n \|a_i\|_2^2$

No. 2.35
Prove that if A and B are two matrices compatible for matrix multiplication, then
(a) $\|AB\|_F \leq \|A\|_F \|B\|_F$
(b) $\|AB\|_F \leq \|A\|_2 \|B\|_F$

No. 2.38
Prove the following.
(a) trace(AB) = **trace**(BA)
(b) trace$(AA^*) = \sum_i = 1^m \sum_j^n |a_{ij}|^2$, where $A = (a_{ij})$ is $m \times n$.
(c) trace(A+B)=**trace**(A)+**trace**(B)
(d) trace(TAT^{-1}) = **trace**(A)