MATH 434/534

Computer Assignment
Due on September 29,2011

Chapter 4
M4.5
Write MATLAB programs to create the following well-known matrices:
(a)
$[A] = \mathbf{wilk}(n)$ to create the Wilkinson bidiagonal matrix $A = (a_{ij})$ of order n:

$$\begin{aligned} a_{ii} &= n - i + 1, \quad i = 1, 2, \ldots, n, \\ a_{i-1,i} &= n, \quad i = 2, 3, \ldots, n, \\ a_{ij} &= 0, \quad otherwise. \end{aligned}$$

(b)
$[A] = \mathbf{Pei}(n, \alpha)$ to create the Pei matrix $A = (a_{ij})$ of order n:

$$\begin{aligned} a_{ii} &= \alpha \geq 0, \\ a_{ij} &= 1 \quad for \quad i \neq j. \end{aligned}$$

(c)
Print the condition numbers of the Wilkinson matrix with $n = 10, 20, 50$, and 100, using the MATLAB function **cond**.

(d)
Fix $n = 20$, and then perform an experiment to demonstrate the fact that the Pei matrix becomes more ill-conditioned as $\alpha \to 1$.

M4.8
The purpose of this exercise is to test that the Hilbert matrix is ill-conditioned with respect to solving the linear system problem.

(i)
Create $A = hilb(10)$. Perturb the $(10, 1)$ entry of A by 10^{-5}. Call the perturbed matrix B. Let $b = rand(10, 1)$. Compute $x = A^{-1}b$, $y = B^{-1}b$. Compute $\|x - y\|$ and $\frac{\|x-y\|}{\|x\|}$. What conclusion can you draw from this?

(ii)
Compute the condition number of both A and B: $Cond(A)$, $Cond(B)$.

(iii)
Compute the condition number of A using the MATLAB command Cond(A) and then use it to compute the theoretical upper bound given in Theorem 4.23. Compare this bound with the actual relative error.

M4.9
Perform the respective experiments stated in Section 4.7 on Examples 4.28-4.30 to show that the eigenvalue problems for these matrices are ill-conditioned.

M4.10

(a)

Write a MATLAB program to construct the $n \times n$ lower triangular matrix $A = (a_{ij})$ as follows:

$$\begin{aligned} a_{ij} &= 1 \quad if \quad i = j, \\ a_{ij} &= -1 \quad if \quad i > j, \\ a_{ij} &= 0 \quad if \quad i < j. \end{aligned}$$

(b)

Perform an experiment to show that the solution of $Ax = b$ with A as above and the vector b created such that $b = Ax$, where $x = (1, 1, \ldots, 1)^T$, becomes more and more inaccurate as n increases due to the increasing ill-conditioning of A. Let $\hat{x}$ denote the computed solution. Present your results in the following form:

n	$Cond(A)$	$\hat{x} = A^{-1}b$	*Relative error* $\frac{\|x-\hat{x}\|_2}{\|x\|_2}$	*Residual norm* $\frac{\|b-A\hat{x}\|_2}{\|b\|_2}$
10				
20				
30				
40				
50				

M4.11

Using MATLAB function **vander(v)**, where v =rand(20, 1), create a 20×20 Vandermonde matrix A. Now take x =ones(20, 1) and $b = Ax$. Now compute $y = A^{-1}b$. Compute y with x by computing $y - x$ and $\|y - x\|$. What conclusions can you draw?